

Index

- C , 58
- C_p , 69
- \mathcal{P} , 18
- \mathcal{P}_i , 18
- \mathcal{R} , 23
- \mathcal{R}^+ , 23
- S_i , 86
- $C(f)$, 19
- $C_i(f)$, 100
- $C(x)$, 87
- E , 17
- $(E, c, \mathcal{S}, n, a)$, 86
- G , 17
- (G, r, c) , 19
- (G, r, c, u) , 93
- (G, r, c, β) , 152
- $M(f)$, 110
- V , 17
- $a_{S,e}$, 86
- c_e , 19
- $c_i(f)$, 21
- $c_P(f)$, 19
- $c_S(x)$, 87
- c_e^* , 27
- \bar{c}_e , 74
- $c_e + \tau_e$, 142
- $d(G, r, c)$, 111, 121
- $d(G, r, c + \tau)$, 142
- $d(v)$, 40
- f , 18
- f_e , 18
- $f^{(i)}$, 99, 103
- f_P , 18
- f^* , 27
- \ln , 24
- r_i , 18
- s_i , 17
- t_i , 17
- x_e , 86
- x_S , 86
- $\lfloor x \rfloor$, 112
- $\alpha(\mathcal{C})$, 61
- $\alpha(c)$, 59
- β , 152
- $\pi(G, r, c)$, 108
- $\rho(G, r, c)$, 22
- $\rho_M(G, r, c)$, 111
- τ_e , 142
- 2 DIRECTED DISJOINT PATHS (2DDP), 134, 146
- Affine. *See* Cost function, linear
- Agent, 18
- Algorithm
 - approximation (*see* Approximation, algorithm)
 - polynomial-time, 12, 28
 - trivial, 122, 142
- Alternating path, 125–126
- Anarchy value, 59–61, 78
 - computation of, 61, 69–73, 78–79
 - lower bounds the price of anarchy, 62–68, 78
 - motivation behind, 59
 - of a cost function, 59
 - of a set of cost functions, 61
 - of a smooth, semiconvex cost function, 60, 78
 - of M/M/1 delay functions, 71–73, 79
 - of polynomial cost functions, 69–71, 78
 - upper bounds the price of anarchy, 61–62, 78
- Approximation
 - algorithm, 44, 121, 133, 146, 165
 - ratio, 44, 121
 - relation to the price of anarchy, 44
 - scheme, 165
- Asymptotic notation, 109
- Atomic. *See* Selfish routing, with atomic agents
- Augmentation, 54
- Balls and bins, 81
- Bandwidth allocation, 83–84
 - proportional sharing, 83
 - with price-anticipating users, 84
 - with price-taking users, 83
- Bicriteria bound, 52, 73–77, 79
 - general version, 76
 - interpretation for additional capacity, 73, 76–77, 79
 - interpretation for improved link technology, 76
 - relation to the π -value of an instance, 108
 - tight examples for, 76
- Braess graphs, 97, 103, 129, 146
 - embedded in a 2DDP instance, 141
 - in Stackelberg routing, 161
- Braess's Paradox
 - can be arbitrarily severe, 129–130
 - formal description, 29–30
 - gives lower bounds on the maximum price of anarchy, 111
 - history of, 13, 46
 - impossibility of algorithmic detection, 132–142, 146–147
 - arbitrary cost functions, 135–141
 - cost functions with bounded incline, 141–142
 - linear cost functions, 133–135
 - polynomial cost functions, 139–140

- using taxes, 142–147
 - in multicommodity networks, 33–34, 46, 114, 118, 147–148
 - in physical networks, 8–9, 14
 - informal description, 5–6
 - relation to strict Pareto inefficiency, 30–31, 46
 - worst-case severity of, 33–34, 122–132, 146
 - arbitrary cost functions, 124–130
 - cost functions with bounded incline, 123, 132
 - linear cost functions, 57–58, 77, 123, 129
 - parameterized by the number of edges removed, 125, 131
 - polynomial cost functions, 123, 131–132
- Calculus and analysis, 11, 15, 43, 45
- Capacity
 - double (*see* Bicriteria bound, interpretation for additional capacity)
 - edge (*see* Edge, capacities)
- Caveats, 9–11, 14
- Centrally controlled traffic. *See* Stackelberg routing
- Chernoff-Hoeffding bounds, 81
- Combinatorial optimization, 12, 15, 41
- Commodity, 17
- Competitive ratio, 44
- Computational complexity theory. *See* NP-completeness
- Cone, 78
- Congestion control, 77
- Congestion games, 115–116
- Conservation constraints, 38
- Control theory, 165
- Convex
 - combination of two points, 23, 45
 - cost flow, 45
 - function, 24, 45
 - function, linear approximation of, 26, 45
 - program, 24, 28, 34, 45
 - semi-, 24, 45, 78
 - set, 23–24, 45
 - vs. semiconvex, 24
- Coordination ratio, 13, 44, 80. *See also* Price of anarchy
- Coping with selfishness, 3, 13
 - by routing some flow centrally (*see* Stackelberg routing)
 - via additional capacity, 52
 - via better link technology, 52
 - via economic incentives, 166–167
 - via edge removals (*see* Braess's Paradox, impossibility of algorithmic detection)
- Cost function, 4, 19
 - affine, 77
 - basic assumptions on, 19, 42
 - concave, 79
 - constant, 42
 - defined for fraction of traffic vs. amount of traffic, 19
 - defined on a restricted domain, 71
 - diverse set, 63–65
 - homogeneous set, 66
 - in classical min-cost flow, 42
 - inhomogeneous set, 66–68, 78
 - linear, 51, 53–58, 77, 111, 112, 123, 129, 133–135
 - linear vs. nonlinear, 145–146
 - linear with no constant term, 54, 77
 - M/G/1, 79
 - M/M/1, 51, 71–73, 79, 148
 - marginal (*see* Marginal cost function)
 - necessity of basic assumptions, 42, 46
 - non-diverse set, 72
 - non-separable, 43
 - polynomial, 69–71, 78, 106–107, 112, 123, 131–132, 139–140
 - quartic (in transportation applications), 70, 78
 - semiconvex, 78, 100, 155
 - set including the constant functions, 62–63
 - strictly increasing, 36
 - used in applications, 43
 - with bounded incline, 123, 141–142
- Cost of a flow, 19
- Cost of a path, 19
- Cut, s - t , 38, 114
- Cycle, 18
- Delay, 8
- Dependencies, 12, 17–18, 52
- Destination (vertex), 17
- Detecting Braess's Paradox. *See* Braess's Paradox, impossibility of algorithmic detection
- Distance labels, 40, 126
- Diverse (set of cost functions), 63
- Double the capacity. *See* Bicriteria bound, interpretation for additional capacity
- Duopoly, 165
- Edge, 17
 - capacities, 42, 93–99, 116, 137
 - oversaturated, 137
 - parallel, 17
 - saturated, 155
 - type, 129

- Edge removal. *See* Braess's Paradox, impossibility of algorithmic detection
- Ellipsoid method (for solving convex programs), 45
- Encoding (of a computational problem), 28, 45
- Existence
of flows at Nash equilibrium (*see* Flow at Nash equilibrium, existence of)
of Nash equilibria in general games, 48
- Facility location, 82, 165
- Fairness, 32–33, 46, 110, 118
- Feasible region, 23
- Fixed point theorems, 48
- Flow
as a point in Euclidean space, 23
as aggregated routes of infinitesimal individuals, 18, 42
at Nash equilibrium (*see* Flow at Nash equilibrium)
average cost of, 19
cost of, 19, 24, 42, 53
cycle, 38–39, 46
directed acyclic, 38–39, 46
feasible, 18
mathematical definition, 18
max-optimal, 111
Nash (*see* Flow at Nash equilibrium)
on edges (*see* Flow on edges)
optimal (*see* Optimal flow)
path, 21
path decomposition of, 18
relation to s - t cuts, 38, 114
- Flow at Nash equilibrium
alternative definitions of, 43
approximate, 89–92, 116
as the limit of Nash equilibria, 48
as the minimum of a convex program, 34–35, 46, 105
characterizations of, 20–21, 34–36, 40–41, 46
computation of, 36, 45–46
convergence to, 10, 14
cost of, 21, 43
defined by shortest-path labels, 40–41, 46
definition, 20
directed acyclic, 39, 46
existence of, 21, 34–35, 42, 46, 48
history of, 43
in capacitated networks (*see* Selfish routing, in capacitated networks)
independence of its path decomposition, 35
inefficiency of (*see* Selfish routing, inefficiency of)
informal description, 20
interpretation as a pure-strategy outcome, 48
relation to an optimal flow, 26
relation to variational inequalities, 36, 46
uniqueness of, 21, 35–36, 46
with linear cost functions, 54, 77
- Flow on edges, 18
characterization of feasibility, 37–38, 46
vs. flow on paths, 42
- Followers (in a Stackelberg game), 151
- Fully polynomial-time approximation scheme (FPTAS), 165–166
- Game theory, 12, 15, 47–48
applications to networks, 12, 165
contrast with selfish routing, 47–48
finite normal form game, 47
- GENERAL NETWORK DESIGN, 132
- Graph, 12, 17, 41
sub-, 58
undirected, 41
- Hardness of approximation. *See* Inapproximability
- Head (of a vertex), 17
- Implementation theory, 167
- Inapproximability, 121–122, 146–147
- Incline, 104–107, 118
definition of, 105
of polynomial cost functions, 106–107
upper bounds the price of anarchy, 105–106
- INCLINE(γ) NETWORK DESIGN, 132
- Inefficiency
of Internet routing, 14
of selfish behavior, 13
of selfish routing (*see* Selfish routing, inefficiency of)
- Inhomogeneous (set of cost functions), 66
- Instance, 19
atomic (*see* Selfish routing, with atomic agents)
Stackelberg, 152
- Interior-point methods (for solving convex programs), 45
- Internet. *See* Networks, computer
- KP model, 80–81
- Latency function, 43. *See also* Cost function
- LCF strategy. *See* Stackelberg strategy, LCF strategy
- Leader (of a Stackelberg game), 151

- Linear approximation, 26, 45, 55
- Linear cost function. *See* Cost function, linear
- LINEAR NETWORK DESIGN, 132
- Link, 17
- Logarithm, 24

- M/M/1 delay function. *See* Cost function, M/M/1
- Marginal cost function, 27, 45, 163
- Marginal cost taxes, 167
- Mathematical programming, 11, 15, 22–26
- Maximum cost, 110–114, 118
 - definition of, 110
 - dependence on the path decomposition, 110
 - in the KP model, 80
 - max-optimal flow, 111
 - price of anarchy, 111–114, 118
- Measure theory, 86, 115
- Mechanism design, 167
- Mixed strategy, 47–48
- Monotone ordering
 - definition, 39–40, 46
 - example, 40
 - existence of, 41, 46–47
- Monotonicity of $d(G, r, c)$ in r , 126–127, 146
- Moral of the story, 5, 6, 26, 59, 71, 73, 77, 107, 131, 133, 155
- Multicommodity, 18

- Nash equilibrium
 - existence in mixed strategies, 48
 - in finite normal form games, 47
 - in selfish routing (*see* Flow at Nash equilibrium)
 - non-existence in pure strategies, 48
 - non-uniqueness, 22, 48, 81
- Nash flow. *See* Flow at Nash equilibrium
- Nash's Theorem, 48
- NCG. *See* Nonatomic congestion game
- Network design, 82–83. *See also* Braess's Paradox, impossibility of algorithmic detection
- Network flow, 12, 15, 37–39, 41, 42, 46
- Networks
 - and game theory, 12
 - capacitated (*see* Selfish routing, in capacitated networks)
 - computer, 7–8, 10–11, 13–14, 77
 - electrical, 9, 14, 58, 77
 - mathematical definition, 17
 - mechanical (*see* Strings and springs)
 - multicommodity, 18
 - nontrivial family of, 68
 - of parallel links, 63–65
 - single-commodity, 17, 36–41
 - transportation, 6–7, 70, 78
 - two-node, two-link, 62–63
 - undirected, 41
 - union of paths, 66–68
 - virtual private, 166
 - with centrally and selfishly controlled traffic (*see* Stackelberg routing)
- Node, 17
- Nonatomic congestion game, 85–88, 114–116
 - action distribution, 86, 115
 - bicriteria bound, 88
 - cost of a strategy, 87
 - cost of an action distribution, 87
 - definition, 85–86
 - elements, 86
 - equilibrium, 87, 115
 - player type, 86
 - price of anarchy in, 87–88, 114
 - rate of consumption, 86
 - relation to selfish routing, 86
 - strategy set, 86
- Nonatomic game, 47–48, 114–115
- Normal form game. *See* Game theory
- NP-completeness
 - coping with, 13
 - informal definition, 11–12
 - references for, 15
 - strong vs. weak, 141, 147
- NP-hard optimization problem, 121

- Objective function, 19, 23, 42, 110
- Online algorithm, 44
- Optimal flow, 20
 - as the minimum of a nonlinear program, 23, 45
 - at Nash equilibrium with respect to marginal cost functions, 26, 27
 - characterizations of, 24–27, 45, 53, 101
 - computation of, 27–28, 45, 155
 - sending additional traffic (*see* Bicriteria bound)
 - unfairness of, 32–33, 46, 110
 - with linear cost functions, 54, 77
- Ordering, f -monotone. *See* Monotone ordering
- Outcome (of a game), 47

- Paradox-free (instance), 135, 139
- Paradox-ridden (instance), 135, 139
- Pareto dominated, 30, 46
- Pareto inefficiency, 13, 46
- PARTITION, 135, 147
- PARTITION, $\frac{1}{3}$ - $\frac{2}{3}$, 163, 166
- Path, 18

- (f, \tilde{f}) -alternating, 125–126
- u - v , 18
- cost of, 19, 42
- decomposition, 18
- shortest, 40
- Payoff, 47
- Performance guarantee (of an approximation algorithm), 121
- Pigou's example
 - as a succinct certificate, 57, 77
 - formal description, 29
 - history of, 13, 46
 - informal description, 3–5
 - nonlinear, 31–32, 46, 107, 152
 - price of anarchy in, 29
 - universality of, 57, 68–69, 77, 78
- Pigou-like example, 59
- Player (in a game), 47
- Polynomial cost function. *See* Cost function, polynomial
- Polynomial-time algorithm, 12, 28
- POLYNOMIAL(p) NETWORK DESIGN, 132
- Potential function, 115
- Potential game, 115
- Prerequisites, 11–12, 15
- Price of anarchy
 - analogues in theoretical computer science, 44
 - bicriteria bound (*see* Bicriteria bound)
 - computation of, 52, 69–73, 78–79
 - connection to the π -value of an instance, 109
 - continuity under perturbations, 78
 - critique of, 11
 - for a diverse set of cost functions, 63–65
 - for a set of cost functions including the constant functions, 62–63
 - for an inhomogeneous set of cost functions, 66–68
 - for most traffic rates, 107–110, 118
 - formal definition, 22
 - history of, 12–13, 44
 - in bandwidth allocation games, 83–84
 - in facility location games, 82
 - in general games, 22, 44
 - in the KP model, 80
 - independence of allowable network topologies, 51–52, 57, 68–69, 77, 78, 84
 - informal definition, 5
 - lower bounds on, 51–52, 57, 62–68, 77–78
 - maximum (*see* Maximum cost, price of anarchy)
 - optimistic, 44 (*see also* Price of stability)
 - precursors to, 44
 - quick-and-dirty bound (*see* Incline)
 - reduces to the anarchy value, 69
 - tight examples for, 51–52, 57, 62–68, 77–78
 - variants on, 44
 - with arbitrary cost functions, 32, 46, 52
 - with concave cost functions, 79
 - with linear cost functions, 31, 51, 53–58, 77
 - with M/G/1 delay functions, 79
 - with M/M/1 delay functions, 51, 71–73, 79
 - with negative or nonmonotone cost functions, 42
 - with nonlinear cost functions, 58–73, 78
 - with polynomial cost functions, 51, 69–71, 78, 106–107
 - with Stackelberg routing (*see* Stackelberg routing)
- Price of stability, 44
 - in network design games, 82–83
 - of selfish routing in atomic unsplittable instances (*see* Selfish routing, with atomic agents)
 - of selfish routing in capacitated networks (*see* Selfish routing, in capacitated networks)
- Pricing. *See* Taxes
- Prisoner's Dilemma, 13, 47
- Probabilistically checkable proofs, 147
- Profit-maximization, 82
- Pure strategy, 47–48
- Queueing delay function. *See* Cost function, M/M/1
- Queueing theory, 71, 79
- Rate. *See* Traffic rate
- Reduction (from one computational problem to another), 147
- Resource allocation. *See* Bandwidth allocation
- Resource augmentation, 79
- Routing
 - distributed and shortest-path, 7–8, 10–11, 13–14
 - Internet (*see* Networks, computer)
 - selfish (*see* Selfish routing)
 - source, 7, 13–14
 - Stackelberg (*see* Stackelberg routing)
- Selfish routing
 - applications of, 6–9
 - assumption of a large population, 18, 20, 42, 80, 99
 - contrast with finite normal form games, 47–48
 - critique of, 9–11

- equivalence with shortest-path routing, 7–8, 13–14
- extensions of the basic model, 43
- formal definition, 17–20
- generalization to nonatomic congestion games (*see* Nonatomic congestion game)
- history of, 12, 43
- in capacitated networks, 93–99, 116
- inefficiency of, 5–6, 13, 29–32
- informal definition, 3
- price of anarchy of (*see* Price of anarchy)
- pure vs. mixed strategies, 48
- stability of, 10, 14
- static vs. dynamic, 10, 14, 43
- strict Pareto suboptimality of, 30–31, 46
- with atomic agents, 42, 48, 99–104, 115–118
 - splittable model, 99–102, 116–117, 148, 166
 - unsplittable model, 103–104, 117–118
- with discontinuous, nonmonotone, or negative cost functions, 42
- with elastic demand, 43
- with incomplete information, 43
- Semiconvex. *See* Convex, semi-
- Shortest-path labels, 40, 126
- Single-commodity. *See* Networks, single-commodity
- Sink (vertex), 17
- Social cost. *See* Objective function
- Source (vertex), 17
- Source-destination pair, 17
- Sparse set (of edges), 124
- Stackelberg games, 151, 165–166
- Stackelberg instance, 152
- Stackelberg routing, 151–166
 - applications, 166
 - examples, 153–155
 - formal definition, 152
 - in multicommodity networks, 161–162
 - in networks of parallel links, 155–159
 - in single-commodity networks, 161
 - limitations of, 153, 155–156, 159–162, 166
 - power of, 153, 155–159, 166
 - reduces the price of anarchy, 151
 - with linear cost functions, 159
- Stackelberg strategy, 152
 - Aloof strategy, 154
 - equilibrium induced by, 152–153
 - flow induced by, 152
 - LCF strategy, 154–159, 162–163
 - optimal, 162–166
 - Scale strategy, 154
 - strong, 161, 162
 - weak, 161
- Strategy
 - mixed, 47–48
 - pure, 47–48
 - Stackelberg (*see* Stackelberg strategy)
- Strategy profile, 47, 115
- Strictly Pareto dominated. *See* Pareto dominated
- Strings and springs, 8–9, 14, 51, 57–58, 77
- Strong NP-completeness, 141, 147
- Subgraph, 58
- SUBSET SUM, 166
- Supply chain, 44
- Tail (of a vertex), 17
- Taxes, 13, 142–147, 167
 - marginal cost, 167
 - relation to edge removals, 142–146
 - that do not contribute to the cost of a flow, 146, 147, 167
- Taylor's Theorem, 45
- TCP (relation to linear cost functions), 77
- Tolls. *See* Taxes
- Topological sort, 41, 46–47
- Traffic rate, 18
 - allowed to vary with network congestion, 43
 - arbitrary vs. unit, 19
 - average-case analysis of, 107–110, 118
 - changing over time, 10, 107
- Transportation. *See* Networks, transportation
- Travel time, 4. *See also* Flow, cost of
- Triangle inequality (of shortest-path labels), 40
- Trivial algorithm, 122, 142
- Type (of an edge), 129
- Undirected graphs and networks, 41
- Union of paths. *See* Networks, union of paths
- Uniqueness
 - essential, 21
 - lack of in general games, 22, 48
 - of flows at Nash equilibrium (*see* Flow at Nash equilibrium, uniqueness of)
- User equilibrium, 43. *See also* Flow at Nash equilibrium
- Utility function, 47
- Variational inequalities, 36, 46, 78
- Vertex, 17
- VPN. *See* Networks, virtual private
- Walk, 18
- Wardrop equilibrium, 43. *See also* Flow at Nash equilibrium
- Wardrop's principle, 43
- Worst-case analysis, 11, 14, 133