

Chapter 1

THE STUDY OF HYDROMAGNETIC TURBULENCE

Although hydromagnetic channel flows used in engineering devices usually are turbulent rather than laminar, we know very little about the structure of turbulent flows in which hydromagnetic effects are significant. The major purpose of this study is to develop an analysis, based on the semiempirical techniques of fluid mechanics, of the internal structure of the turbulent hydromagnetic flow of an incompressible fluid in a high-aspect-ratio rectangular channel subjected to a d-c transverse magnetic field. A secondary purpose is to indicate that, under certain conditions, the solutions obtained for the d-c turbulent flow should be applicable, with minor modifications, to induction-driven flows. The principal results of this study consist of the figures in Chapter 7, giving normalized mean velocity profiles, current distributions, and velocity correlations for the d-c turbulent flow over a range of operating conditions. These provide a basis for both a qualitative and a quantitative picture of the internal structure of the flow.

Even for laminar hydromagnetic flows, only a few mathematical solutions have been obtained. All of these are for conditions closely related to one of the approximately half-dozen general situations in which useful solutions have been obtained for laminar hydrodynamic flows. Those that are available, however, are sufficient to give at least a qualitative picture of most laminar hydromagnetic flows of technical interest, and the electromechanical interactions in such flows have been well understood for some time. With this background plus the widely held conviction that turbulent flows inevitably are accompanied by higher power losses than laminar flows, we might be tempted to conclude that, as a practical matter, we should try to avoid turbulence and design all devices for laminar flow. A quick survey of hydraulic equipment, however, suggests that such a conclusion would be erroneous. Except for the laminar films always present on solid boundaries, laminar flow in hydraulic equipment is a rarity; viscometers and hydrodynamic bearings probably are the only technically important applications. It is true that, for the flow geometries, fluid properties, and pressure gradients or flow rates present in most hydraulic equipment, laminar flow would yield lower power losses than turbulent flow, if it could exist. But this fact is largely irrelevant to design activity because the nature, laminar or turbulent, of a hydrodynamic flow is determined by a single parameter, the Reynolds number, and cannot be selected independently of

the other flow conditions. The design of power-level equipment for laminar-flow conditions apparently always yields bulkier, more complex configurations and/or higher total power losses than reasonable turbulent-flow designs. There is yet no reason to believe that this experience of hydraulic engineers will be reversed in the design of hydromagnetic equipment.

The study of hydromagnetic turbulence, however, is interesting for reasons quite apart from those associated with the design of hydromagnetic machines. A clearer understanding of turbulent hydro-magnetic flows will result in greater insight into strictly hydrodynamic turbulence and into the mechanism of transition between laminar and turbulent flow regimes. The addition of the magnetic field provides a parameter that can be varied at will to change the loss mechanism and the distribution of losses across the flow. The plasma dynamicists reportedly have encountered turbulence-like activity in plasmas with very high energy densities in devices such as the "Stellarator." This device represents one application where the onset of turbulence is a disaster causing rapid destruction of the plasma. The continuum formulation outlined in Chapter 3 and used in this study is too crude to yield detailed predictions of the behavior of plasmas in strong magnetic fields. It does appear, however, to present possibilities for a spontaneous turbulent interaction between velocity and magnetic-field fluctuations in an essentially stationary fluid, when the magnetic field and electrical conductivity become sufficiently high.

Attempts to gain some understanding of hydromagnetic turbulence have taken several forms. Some work has been done, notably by Batchelor² and Chandrasekhar,⁴ toward extending the statistical theory of turbulence to hydromagnetic flows. Unfortunately, this theory is still less developed than the statistical theory of hydrodynamic turbulence, which cannot yet treat effectively any technical flow. Lock¹⁰ has attempted by use of perturbation theory to compute the critical Reynolds number at transition between laminar and turbulent flow for hydromagnetic flow between two parallel planes in the presence of a d-c transverse field. Stewart¹⁴ has performed similar calculations for the flow with a longitudinal magnetic field. To date, these results have not been verified experimentally; Lock's critical Reynolds numbers exceed by a factor near 200 those obtained experimentally by Murgatroyd.¹² These large discrepancies have been attributed to the facts that Lock's results refer to the instability of laminar flow in the presence of only infinitesimal disturbances, while Murgatroyd's apply to the damping out of turbulence in the presence of fairly large disturbances. The critical Reynolds numbers for these two phenomena can differ by large factors in hydrodynamic flows and, probably, in hydromagnetic flows also.

For our present study, the most useful past work consists of two sets of experiments, one by Hartmann and Lazarus⁷ performed in 1937 and one by Murgatroyd¹² performed in 1953, in which measure-

ments were made of pressure gradient, flow rate, and magnetic flux density on flows of mercury in rectangular channels subjected to d-c transverse magnetic fields. The results of these two sets of experiments, which are described in detail in Chapter 5, exhibited both similarities and differences that have been informative in the present study. There have been, of course, previous efforts to explain or correlate the trends shown in these experiments from some theoretical bases. The two analyses accompanying the presentations of the data were restricted to the correlation of the pressure drop versus flow data, and were based on rather unsatisfactory applications of dimensional analysis. Hartmann and Lazarus attempted to separate their results into a "turbulence-damping effect," unique to turbulent flows, and a "viscosity effect," similar to that found in laminar flows. The procedures by which they effected this separation, however, involved several arbitrary decisions and were based on a rationale implying that the turbulence-damping effect is independent of the electrical conductivity of the fluid. As Murgatroyd pointed out, there is no justification, either from theory or experiment, for this assumption. Murgatroyd proposed instead a correlation scheme based on the assumption that the friction factor f (defined in Chapter 2) for the turbulent hydromagnetic flow is independent of the fluid viscosity. This assumption also is untenable. Although turbulent flows exhibit a tendency for the local structure in the central portion of the flow to be independent of fluid viscosity, the friction factors for channel flows always depend strongly on fluid viscosity. If the viscosity is denoted by η , the friction factors in laminar hydrodynamic and hydromagnetic flows tend to vary as η and $\sqrt{\eta}$, respectively, while $1/\sqrt{f}$ for turbulent hydrodynamic flow tends to vary linearly with $\log \eta$.

Both Murgatroyd¹² and Hartmann and Lazarus⁷ realized that the theoretical portions of their papers were at best incomplete, and indicated that they were carrying on further work that would be published in the future. These works have not yet appeared, however, and the phenomena exhibited in the published experiments have remained largely unexplained.

The objectives of the present study are somewhat more ambitious than those of the theoretical efforts just described, for we wish not only to provide a correlation scheme that will relate the two sets of experimental data available but also to use these data to deduce the internal structure of the flow and thereby gain a better understanding of the details of the interactions within the flow. The approach used here is based on the suppositions that turbulent hydrodynamic and hydromagnetic flows should be closely related, much as the corresponding laminar flows, and that the techniques that have proven most fruitful in the analysis of turbulent hydrodynamic flows might also permit a successful attack on the turbulent hydromagnetic flow. Thus, we are led to apply the "semiempirical" techniques of fluid mechanics to the present problem. These combine the use of the

basic hydromagnetic equations, the available experimental data, and the logic of dimensional analysis, drawing from each all the information that can be obtained simply.

Because these techniques and the results obtained by their use in hydrodynamics are unfamiliar to many electrical engineers, we describe in Chapter 2 the basic method of analysis used, develop the accepted formulas for velocity profiles, velocity correlations, and friction factors in turbulent hydrodynamic flows, and discuss the agreement between these theoretical predictions and parts of the vast body of supporting experimental data that has accumulated in the past thirty years.

In Chapter 3, we turn to the basic equations governing incompressible hydromagnetic flows. This chapter treats three topics. In the first section, we write the basic equations, select a normalization scheme, and derive several useful forms for the normalized equations. The second part consists of two sections in which we develop the boundary conditions for the flow with a d-c transverse magnetic field and also a classification, which turns out to be valid for both the d-c and a-c flows, of all the interesting flow variables according to their even or odd spatial symmetry. The third part consists of the derivation of a partial solution in which we express the mean velocity and mean perturbation field in terms of the parameters of the system and definite integrals of two statistical correlations. The principal results of this part, however, are the conclusions that the normalized mean velocity can be approximated by a function of M , R^* , and ξ only, where M is the channel Hartmann number, R^* the Reynolds number, and ξ a normalized distance, and that the normalized perturbation field can be approximated by the product of R_v and another function of M , R^* , and ξ . Here R_v is the magnetic Reynolds number. The worker in magnetohydrodynamics will recognize these as approximations suited to simple flows with small magnetic Reynolds numbers in which the perturbation fields are proportional to R_v but so small that they have little effect on fluid motion.

In Chapter 4, we combine the conclusions of Chapter 3 with the analytical techniques of Chapter 2 to gain the simplification obtainable from the logic of dimensional analysis. In this process, we introduce as assumptions the general statements of two empirical laws of hydrodynamic channel flows. These are (1) that at points near the boundaries, the mean-velocity profile is independent of the channel span and (2) that at points near the center, the local structure of the mean flow is independent of fluid viscosity. The first assumption implies that the flow near the boundary looks like a semi-infinite flow past a single surface; the second implies that viscous dissipation in the central part of the channel is negligible. Both of these assumptions are valid for turbulent hydrodynamic flows and for laminar hydromagnetic flows with high Hartmann numbers. Their final justification here, however, rests with the end results of the study. The

principal results of Chapter 4 are formulas giving the velocity profile and friction factor in terms of two unknown functions. One of these depends on M^2/R^* (which does not involve fluid viscosity) and gives the distortion in the velocity profile caused by hydromagnetic effects. The other depends on M/R^* (which does not involve the channel span) and represents, over the central part of the channel, a constant addition to the mean velocity caused by hydromagnetic effects.

The theoretical analyses presented in Chapters 3 and 4 reduce the problem of finding the normalized mean-velocity profile from the determination of a single function of four variables, M , R^* , R_v , and ξ , to the determination of two functions, each dependent on a single variable. We must turn to the experimental data for further simplification. The two sets of experimental data used here are described in some detail in Chapter 5. Chapter 6 is devoted to the correlation of theory and experiment. Fortunately, the common characteristics exhibited by the data give good reason for neglecting one of the unknown functions and provide an empirical determination of the other. The close fit, when plotted properly, of points derived from all the experimental data for highly turbulent flows to a single function of M^2/R^* represents an experimental justification of the preceding theory. The data also suggest a method for computing an analytic approximation to this unknown function. The assumptions on which this approximation are based, however, are not valid at low values of M , and the result consequently differs from the empirical curve by a nearly constant value for moderate and large M^2/R^* . This approximation does serve, nevertheless, as a check on the internal consistency of our results.

There is some difficulty, discussed fully in Chapter 6, in the correlation with theory of the data obtained from flows that were almost laminar. In effect, we are forced to admit that there is no sharp boundary between laminar and turbulent hydromagnetic flows, and that our simplifying assumptions do not always conform to the complex reality of flows that are neither laminar nor very turbulent. Hartmann⁶ long ago provided a good analysis of the laminar flow; the present study appears to provide a satisfactory treatment of flows that are highly turbulent. But there still exists a small range of conditions near transition where neither analysis is really valid. This situation is not surprising, however, for the same condition still exists in the analysis of the simpler hydrodynamic flows.

Once all the unknown functions are eliminated from the analysis, we can easily compute mean-velocity profiles, approximate current distributions and velocity correlations, and, if we wished, the mean perturbation fields from the mathematical formulas in Chapters 3 and 4. Chapter 7 contains results of these calculations together with a discussion of some interesting trends occurring at high values of M^2/R^* . These curves are the essential results of the study.

Chapters 8 and 9 constitute a short discussion of laminar and tur-

bulent induction-driven flows. They consist primarily of mathematical analyses based on the hydromagnetic equations set forth in Chapter 3. The major qualitative conclusion derived from these chapters is that, under reasonable operating conditions, the velocity and electromagnetic-field distributions in the induction-driven flows are analogous to those in the corresponding d-c flows. By "reasonable operating conditions," we refer here to situations in which the electrical slip frequencies seen by the moving fluid are sufficiently low that the electrical skin effect is negligible, yet sufficiently high that the zero-average double-frequency components of forces acting on the fluid are effectively filtered out by fluid inertia. When these conditions are satisfied, the transverse magnetic field is uniform across the channel, and the distribution of effective forces acting on the fluid is similar to that in the corresponding d-c flows. Then, the variations in mechanical quantities, such as mean velocity and wall shear stress, in the a-c flow are similar to those found for the corresponding d-c flow, and variations in electrical quantities, such as magnetic flux density, electric field, and current density, differ from those found for the d-c flows mainly by a superposed traveling-wave modulation.

In the induction-driven flows, the magnetic Reynolds number always is a very important variable; the operation of every induction machine depends on the interaction of the fields associated with the electrical excitation and the reaction fields associated with the currents in the moving material. The latter, of course, are the analog of the perturbation fields in the d-c flow, except that they cannot be small if there is significant energy conversion in the flow. Though often large, the effect of magnetic Reynolds number is quite simple, and it appears in our analysis as a variation, with average slip and channel reactance-to-resistance ratio, of the total transverse magnetic field and, therefore, of the effective Hartmann number for the flow.

The analysis in Chapter 8 generally follows lines laid down by Blake in a paper³ on the design of an induction pump, except that it discusses more fully the mechanical aspects of the flow. The last section of this chapter is a digression in which several well-known analogies between the operation of induction pumps and conventional induction motors are related to the present analysis.

Chapter 9 is similar in content to Chapter 3. Here we show that the equation determining the mean-velocity profile in the induction-driven flow takes a form nearly identical to that governing the velocity profile in the d-c flow. The apparent implication of this result is that the curves for d-c flows presented in Chapter 7 can be adapted, by minor modifications, for application to a turbulent induction-driven flow.

The last chapter, Chapter 10, contains a summary of conclusions derived from this study and several suggestions for interesting future work.