BEYOND THE SCHELLING’S SEGREGATION MODEL: IS IT EQUIVALENT TO BE REPULSED BY DISSIMILAR RATHER TO BE ATTRACTED BY SIMILAR?

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Abstract

From the Schelling model of segregation, we derive models of group formation that shed light on segregation or mixing patterns observed in spatial grid networks. Individuals have types and see type-dependent benefits or drawback from theirs neighbours: this leads each one to be attracted or repulsed by its own like or unlike. This framework allows to studying many spatial phenomena that involve individuals making location choices as a function of the characteristics and choices of their neighbours. Our goal is to grow social structures in silico and to ask if related micro-specifications generate similar macro-phenomena of interest. Regarding (i) the amount of segregation-mixing, (ii) the congruence between micro-motives and macro-behaviour and (iii) the nature of frontier between clusters, we examine the properties of the steady-state equilibrium.

Introduction

This article is based on the Schelling’s checkerboard model of residential segregation?: this model has become one of the most cited and studied models in many domains as economics, sociology, complex systems science… ? ? ?. In the Schelling model the world is a 2-dimensional grid composed either by locations occupied by agents or by vacant ones. The perception of an agent is centred on its local neighbourhood only. There are two types of agents. The behaviour of one agent consists to move away on if it is not satisfied. Satisfaction is relative to the proportion of agents with a dissimilar type in the neighbourhood.

In this paper, the goal is to grow social structures in silico and to ask if related micro-specifications generate similar macro-phenomena of interest. We define a generic framework in order to shape several models with various micro-motives. This family of models allows us to show that a wide variety of macro-behaviours can emerge despite the fact that they come from a same mould and share some common features. In particular, we propose to consider the following issues: Q1) Is it equivalent to flee regarding the proportion of dissimilar agents among the neighbourhood agents rather than the number of dissimilar agents in the full set of neighbours? Q2) Is it equivalent to be repulsed (resp. attracted) by dissimilar neighbours rather to be attracted (resp. repulsed) by similar neighbours?

The AR models

To answer these questions, we define the family of attraction-repulsion models (AR models). For all these models, the world is a 2-dimensional grid where nodes-cells represent either locations occupied by agents or vacant ones. A vacant cell could be occupied by an agent later. The perception of an agent is centred on its local neighbourhood only. One assumes that the neighbourhood of an agent is constituted both by the nearest agents and the vacant cells surrounding him: in this paper we consider the Moore neighbourhood composed of the eight nearest cells surrounding it. Let be V the set of vacancy cells and A the set of agents. We assume that the number of agents (#A) is conserved and the total volume in which they move (#A + #V) is constant. The density of agents is the ratio \( d = \frac{#A}{#A + #V} \). There are two types of agents and each agent has its own type. The agent’s type can never change. For convenience we will denote by a color, blue and yellow, the two possible types. Yellow and blue agents can be interpreted as individuals representing any two groups in society (two genders, smokers and non-smokers, etc.). Let B (resp. Y) the set of agents in the blue type (resp. yellow type). So, \(#B + #Y = #A\) and, at the global level, the basic hypothesis is \((#B = #Y)\). Each agent is satisfied or unsatisfied according to its own type and the type of its neighbours.

Micro motives

In the Schelling model one agent moves if its utility \( u \) falls below a certain threshold \( (u \leq \tau) \). In this section we generalize this model in considering different utility functions and we propose to study the two cases where the utility is either below or above the threshold \( \tau \).

Utility functions For each agent \( a_i \), considering only the agents in its neighbourhood, we define the two utility functions:

\[
\Delta_i = \begin{cases} 
\frac{\delta_i}{\sigma_i + \delta_i}, & \text{if } \sigma_i + \delta_i \neq 0 \\
1, & \text{else}
\end{cases}
\] (1)
where $\delta_i$ (resp. $\sigma_i$) is the number of dissimilar (resp. similar) nearby for the agent $a_i$.

In the same way, considering the set of all the cells in the neighbourhood of the agent, we define the two utility functions:

$$\Delta_i' = \frac{\delta_i}{\text{neighbourhoodSize}}$$

$$\Sigma_i' = \frac{s_i}{\text{neighbourhoodSize}}$$

Let’s note that $\Delta_i + \Sigma_i = 1$, while, in general, $\Delta_i' + \Sigma_i' \neq 1$.

So, given a threshold $\tau$ in the range $[0, 1]$, there are potentially eight ways to express a condition to be satisfied (Table ??). Let’s note that the model $M_0$ corresponds to the Schelling model of segregation.

### Table 1: Potential conditions to be satisfied

<table>
<thead>
<tr>
<th>Model</th>
<th>$u \leq \tau$</th>
<th>Model</th>
<th>$\tau \leq u$</th>
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<tbody>
<tr>
<td>$M_0$</td>
<td>$\Delta \leq \tau$</td>
<td>$M_1$</td>
<td>$\Delta \leq \tau$</td>
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<tr>
<td>$M_1$</td>
<td>$\Delta \leq \tau$</td>
<td>$M_5$</td>
<td>$\Sigma \leq \tau$</td>
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<tr>
<td>$M_2$</td>
<td>$\Sigma \leq \tau$</td>
<td>$M_6$</td>
<td>$\tau \leq \Sigma$</td>
</tr>
<tr>
<td>$M_3$</td>
<td>$\Sigma \leq \tau$</td>
<td>$M_7$</td>
<td>$\tau \leq \Sigma$</td>
</tr>
</tbody>
</table>

### Complement model

Two models $M_i$ and $M_j$ are said to be complementary iff the conditions to be satisfied are logical complementary; then, we note $M_j = M_i^\perp$.

Obviously the equation $\overline{M_i} = M$ holds. We have $M_4 = M_6$, $M_5 = M_1$, $M_6 = M_2$ and $M_7 = M_3$.

### Dual model

Two models $M_i$ and $M_j$ are said dual iff the conditions to be satisfied are dual; we note $M_j = M_i^\ast$. We say that two conditions $(U_i \leq \tau_i)$ and $(\tau_j \leq U_j)$ are dual iff $(U_i, U_j) \in \{(\Delta, \Sigma), (\Sigma, \Delta), (\Delta', \Sigma'), (\Sigma', \Delta')\}$.

Obviously the equations $\forall i, M_i^\ast = M_i$ and $M_i^\perp = M_i^\perp$ hold. More, as $\Delta + \Sigma = 1$, $M_0^\ast = M_0$ and $M_2^\perp = M_2$. Finally, we have $M_4 = M_2^\ast = M_2$, $M_5 = M_3^\ast = M_3$, $M_6 = M_0^\ast = M_0$ and $M_7 = M_1^\ast = M_1$.

### AR models

All this leads us to consider six models of segregation/mixing only. For each model, Table ?? gives the local condition under which an agent is satisfied. Let’s note that all the AR models are defined from the two models $M_0$ and $M_1$ only.

### Table 2: AR models: condition to be satisfied

<table>
<thead>
<tr>
<th>Model</th>
<th>$\Delta \leq \tau$</th>
<th>Model</th>
<th>$\tau \leq \Sigma$</th>
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<tbody>
<tr>
<td>$M_0$</td>
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<td>$\Delta \leq \tau$</td>
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<td>$\Sigma \leq \tau$</td>
<td>$M_7$</td>
<td>$\tau \leq \Sigma$</td>
</tr>
</tbody>
</table>

### Rules governing agents movement

In the standard Schelling’s models, as soon as an agent is unsatisfied, it moves to another place where it becomes satisfied. So, the local behaviour is an optimisation process which needs any agent to access information about any cell-location in order to compute its utility function in the target cell.

To stay is in the spirit of the complex systems paradigm, in AR models there is no decision-rule to decide whether or not an agent gains an advantage by means of migration towards a target cell. To find a new place, an unsatisfied agent uses a simple rule (what we call the Eulogy to Fleeing (EF) rule): a location is randomly chosen from the world and the agent moves into it if and only if this location is vacant. Consequently, an agent may move at random towards new locations by allowing utility-increasing or utility-decreasing moves. As the moves do not equate to immediate benefits, it is challenging to predict the overall emerging effect.

### Macro behaviour

For the six AR models, simulations will show that, applying the EF rule, in many cases, the population reaches a state where all the agents are satisfied or, at least, a large proportion of agents are satisfied. In the first case we will say that there is convergence and in the second case quasi-convergence. In this paper, we do not discuss the conditions which guarantee convergence towards equilibrium or quasi-equilibrium; we select system conditions in which one of these two cases occurs. Let’s note that the EF rule has already been used within Schelling’s models, leading the system towards equilibrium ?.

### Segregation vs. mixing

According to its micro-motive, we may assume that each AR model leads at the global level of the population either towards segregation or mixing. For a given model, regarding if the micro-motive is based on repulsion or attraction in the face to similar or dissimilar neighbours, we indicate if the macro-behaviour should be rather either segregation or mixing (Table ??). Given to models $M_i$ and $M_j$:

- if $M_j$ and $M_i$ are complementary, then the respective macro-behaviour and micro-motives are opposed but the target neighbours are identical.
- if $M_j$ and $M_i$ are dual, then the respective macro-behaviour are identical but the micro-motives and target neighbours are opposed.
Because the $M_0$ (resp. $\overline{M_0}$) model is self-dual, it can be view either as a model of repulsion against its dissimilar (resp. similar) agent-neighbours or a model of attraction for its similar (resp. dissimilar) agent-neighbours; in both case there is emergence of segregation (resp. mixing).

An index to measure the segregate-mixing ratio In order to gain some insight into the segregation-mixing level, it is necessary to measure the global state of segregation of the world. We reformulate measures proposed by $\tau$, $\overline{\tau}$ and $\overline{\tau}$.

First, for each time $t$, we define a global measure of similarity as:

$$s(t) = \frac{1}{\#A} \sum_{i} \#A(1 - \Delta_i(t))$$

Then we define the segregate-mixing index by:

$$segMix = \begin{cases} \frac{s - s_{\text{rand}}}{1 - s_{\text{rand}}} & \text{if } s \geq s_{\text{rand}} \\ \frac{s - s_{\text{rand}}}{s_{\text{rand}}} & \text{else} \end{cases}$$

where $s_{\text{rand}}$ is the expected value of the measure $s$ implied by a random allocation of the agents in the world. So, a zero $segMix$ index corresponds to a random positioning of the agents. The maximum value of 1 corresponds to a configuration with two homogeneous patterns (complete segregation into two same-colour groups), whereas negative values point towards highly mixed populations.

An index to measure the congruence between micromotives and macro behaviour We define the micro-Macro index as:

$$mM = \begin{cases} \overline{u} \overline{\tau} & \text{if } \tau \geq \overline{\tau} \\ 1 - \overline{u} \frac{1 - \overline{\tau}}{1 - \tau} & \text{else} \end{cases}$$

where $u$ is the utility function and $\overline{\tau} = \text{mean}_i(u_i)$. The value 1 corresponds to the theoretical optimal case where dynamics build exactly the needed liveable configurations: in this case the congruence between micro and macro levels is maximum. The value 0 corresponds to the extreme case where dynamics build much more liveable configurations than necessary to reach the emergent macro-state: in this case there is no congruence between micro and macro levels.

Frontier A frontier is a generic concept that has different instantiations depending on the context in which it is considered. A common class of frontiers is found in the geographical domain, where they appear as fronts. The interest we take in this concept lies on two aspects: from a static standpoint, a frontier enables the separation of incompatibilities; however, as it allows at the same time some form of communication between them, a dynamic perspective is also relevant. So, we consider a frontier as a structure which both determines the “borderland” between two aggregates of opposite types and allows communication between them.

Definition A frontier is composed of the cell-locations where contact occurs between two dissimilar agents. We consider contacts as being of two types: direct or indirect. A direct-contact refers to agents being directly linked, whereas an indirect-contact is mediated through a vacant location. In the real world, a direct-contact can be exemplified through the contact of a healthy person with a person having a communicable disease, whereas an indirect-contact is achieved through some intervening medium e.g. air. Let $D$ (resp. $I$) the set of direct (resp. indirect) contacts:

$$D = \{(a_i, a_j) \in B \times Y | a_j \in N(a_i)\}$$

$$I = \{(a_i, a_j, v) \in (B \times Y \times V) | v \in N(a_i) \cap N(a_j)\}$$

where $N(a_k)$ is the neighbourhood of the agent $a_k$. Then we define the frontier as: $F = (A_F \cup V_F, E_F)$; where $A_F$ is the subset of agent-cells that are at least one coordinate of an element of $D$ or $I$, $V_F$ is the subset of vacant cells that are at least one coordinate of an element of $I$ and $E_F$ is the set of links between neighbouring cells of $F$.

Characteristics of a Frontier To enable consistent comparisons of frontiers, we take into account their importance relative to the entire world and their openness. Thus, we define what we mean by a frontier’s occupancy and porosity. These two criteria are chosen to address the ambivalence between separation and exchange, as the main characteristic of a frontier.

We define the occupancy as the ratio between the number of cells forming the frontier and the total number of cells in the world:

$$\text{occupancy}(F) = \frac{\#(A_F \cup V_F)}{\#A + \#V}$$

For instance, if each agent is placed on a checkerboard according to its type\(^1\), all the agents are on the frontier and so the occupancy is equal to 1.

In Material Physics, porosity is a measure of how much of a rock is open space in between spores or within cavities of

\(^1\)On a checker-board, a yellow agent is on a black square while a blue agent is on a white square, so there are no vacant places.
the rock: it is defined as the ratio of the occupancy of voids in a material to the occupancy of the whole. By analogy, we consider the elements of $D$ as representing the voids in a material (a lack of communication impediments). We define then the porosity as the proportion of direct-contacts: 
\[
\text{porosity}(F) = \frac{\#D}{\#D + \#I}
\]

Simulation Framework

Experiments are performed via the NetLogo multi-agent programmable modelling environment. The pseudocode for simulating AR models is defined in algorithm 2. by instantiating the utility function via the update satisfaction method we obtain the six simulators that we experiment with.

Simulations are performed on a $L \times L$ lattice of locations: $L$ is set to 50 and the density of agents $d$ is 90% which are standard values to simulate the Schelling model. The agent set is positioned in a random initial configuration, such that the vacant locations and the two types of agents are well mixed and the segMix index is close to 0.

Algorithm 1 Simulation of the AR models

1. $t \leftarrow 0$, $density \leftarrow d$, $threshold \leftarrow \tau$
2. create a grid network and position at random the agents on it
3. update the satisfaction of all the agents at time 0
4. while not (all the agents are satisfied) do
5. for each agent $a_i$ do
6. if not ($a_i$ satisfied) then
7. choose a node-location at random on the grid
8. if the location is vacant then
9. $a_i$ moves to this location
10. end if
11. end if
12. end for
13. $t \leftarrow t + 1$
14. update satisfaction of all the agents at time $t$
15. end while

Models of segregation

In this section we consider among the six AR models the ones which lead to segregation. In order to compare the resulting shapes we will use (i) the segMix index, (ii) the mM congruence index and (iii) the characteristics of the frontier which emerges between clusters of agents with a same type.

$M_0$ model

In the Schelling’s segregation model, agents are satisfied if the proportion of dissimilar neighbours among the agent-neighbours is below a threshold and unsatisfied if the proportion of similar neighbours is above this threshold. The Schelling’s model is a particular case among the AR models: it corresponds to $M_0$. To compute satisfaction, we consider the condition ($\Delta \leq \tau$) where $\tau$ can be interpreted as the tolerance of the agents against their dissimilar neighbours. If $\tau \ll 0.5$, agents are rather intolerant, else if $\tau \gg 0.5$ agents are rather tolerant.

Because the smallest step in the fraction of satisfied neighbours is $1/8$, we consider the two examples with $\tau = 3/8$ and $\tau = 5/8$: if $\tau = 0.375$, agents are rather intolerant: one agent $a_i$ will be satisfied in eighteen cases. If moreover the agent has exactly eight nearby agents, it cannot suffer more than three dissimilar neighbours. At the opposite, for the value $\tau = 0.625$ all the individuals are rather tolerant and, if moreover an agent has exactly eight nearby agents, it can suffer at most five dissimilar agents in its vicinity.

segregation Because $M_0$ is self-dual, the dynamics can be interpreted either as a phenomenon of repulsion against dissimilar neighbours or attraction for similar neighbours. From the first point of view $\tau$ is the tolerance against dissimilar while in the second case $(1 - \tau)$ is the appeal for similar. Simulations show that both intolerant and tolerant agents lead the population to segregate with, of course, an segMix index higher for intolerant agents ($0.952$ vs. $0.519$) (figure ??). As we use the ER rule, this result confirms the unexpected behaviour provided by the Schelling’s model where, in spite of their tolerance, to some extend, agents tend to group together by affinity.

Micro-Macro congruence The gap between the threshold of tolerance and the mean utility over the whole population is surprisingly high at the end of a run. If agents are intolerant ($\tau = 0.375$) the congruence between micro and macro levels is close to 0 ($mM = 0.07$); in this way, complex dynamics build much more liveable configurations than necessary. The fact that tolerant agents ($\tau = 0.625$) tend nevertheless to group together by affinity can be explained by a low micro-macro congruence ($mM = 0.37$).

Frontier If agents are intolerant, the emerging frontier is essentially built with vacant cells (figure ??: white squares). There are many indirect-contacts and homogeneous patterns are isolated by a no-man’s-land of vacant-cells. As tolerance increases the no-man’s-land shape becomes more and more complex: as in a real landscape when roughness dictates many meanders to the edge of a lake, the complexity of contours increases (figure ??). We observe that both the occupancy and the porosity increase with tolerance. So, both the shape and the composition of the frontier change as agents become more and more tolerant.

$M_1$ model: repulsion for the unlike

The $M_1$ model is a variant of the $M_0$ model where agents are satisfied if the normalized number of dissimilar agent-
The dynamics can be interpreted as a phenomenon of repulsion against dissimilar neighbours. Once again, this should lead the system to converge toward a global configuration with segregation. Indeed, we can observe that both intolerance (figure ??) and tolerance (figure ??) lead to segregation. Nevertheless, compared to the $M_0$ model, the phenomenon of segregation is less marked (table ??); more, if agents are rather tolerant ($\tau = 0.625$), this tendency to group together by affinity is much lower than in the $M_0$ model ($\text{segMix} = 0.3$ vs. $\text{segMix} = 0.519$).

**Micro-Macro congruence** For intolerant, as well tolerant, agents the gap between the threshold and the mean utility over the whole population is much lower than in the $M_0$ model (e.g. $mM = 0.23$ vs. $mM = 0.07$).

**Frontier** As tolerance increases, there is always a tendency to transform no-man-land frontier to space-fill curve; but here the frontier occupies more places in the world and its porosity is higher (figure ??).

$M_1$ model: attraction for the like

As the $M_1^*$ model is the dual of $M_1$, micro-attraction against similar agents-neighbours induces macro-segregation. Indeed, agents are satisfied if the normalized number of similar agents-neighbours is above the threshold: so, $\tau$ represents the appeal of one agent for its similar neighbours. Let’s note that the more the appeal the more the incentive to move is. As, at the local level, there is attraction for the cells which share the same type, once again, this should lead the system to converge toward a global configuration with high segregation. Let’s remark that in order to compare the macro behaviour between models $M_1$ (or $M_0$) and $M_1^*$, we have to consider tolerance $\tau$ with appeal $(1 - \tau)$ (e.g. 0.375 vs. 0.625).

**segregation** The emerging segregation can be interpreted as homophily, that is the tendency of agents to segregate in spatial groups with similar others. First of all, let’s note that the system converges towards global satisfaction if $\tau = 0.375$; but, with $\tau = 0.625$, it remains forever about 10% of unsatisfied agents in the population; so there is quasi-convergence only. As expected, we can observe that high appeal (e.g. $\tau = 0.625$) leads to a strong segregation (e.g. $\text{segMix} = 0.91$) (figure ??); but, more surprisingly, low appeal (e.g. $\tau = 0.375$) leads also to some extent to an...
segregate population of agents ($\text{segMix} = 0.567$) (figure ??) comparable to what we obtain for the $M_0$ model with tolerant agents.

**Micro-Macro congruence** For appealing agents the gap between the threshold and the mean utility over the whole population is much lower than in the $M_0$ model ($mM = 0.22$ vs. $M = 0.07$); results are rather comparable to what we obtain for the $M_1$ dual model ($mM = 0.22$ vs. $mM = 0.23$).

The fact that agents weakly attracted by similar neighbours segregate nevertheless in spatial group with similar others can be explained by a low micro-macro congruence ($mM = 0.37$) comparable to what we obtain for the $M_0$ model with tolerant agents.

**Frontier** When the strength of attraction towards similar neighbours is high, the frontier have stable properties (i.e. occupancy and porosity are quasi-invariant). Moreover, these properties are comparable to what we obtained with the $M_0$ model (Table ??). Nevertheless, due to its permanent dynamics, the frontier have now a radically different shape: there is a certain thickness inside which the cells move infinitely (figure ??).

In this section we consider the models which lead to mixing among the AR models. In these cases it is useless to look at the frontier because, at convergence, this one occupies the world in totality (i.e. occupancy $\approx 1$) and porosity is very high (i.e. porosity $\approx 0.9$)

**$M_0$ model**

In this model agents are satisfied if the proportion of dissimilar neighbours is above $\tau$ and unsatisfied if the proportion of similar neighbours is below $\tau$. Let’s remember that this model is self-dual. Because it is the complement of $M_0$, macro-mixing is induced either by micro-repulsion against similar neighbours or micro-attraction by the agent-neighbours which share the same type.

**Mixing** We can observe indeed that high value of $\tau$ leads to a low value of $-0.53$ for the $\text{segMix}$ index which reveals a strong mixing: the population is mainly an alternation of homogeneous lines or columns constituted by agents with the same type (figure ??). As $\tau$ decreases, the $\text{segMix}$ index increases too and so mixing decreases (figure ??).

**Micro-Macro congruence** Whatever the threshold is, the gap between the threshold and the mean utility over the whole population is relatively low at the end of a run ($mM \approx 0.64$).

**$M_1$ model: attraction for the unlike**

In the $M_1$ model agents are satisfied if the normalized number of dissimilar agent-neighbours is above the threshold. Because this model is the complement of $M_1$, micro-attraction for dissimilar neighbours will induces macro-mixing.

**Mixing** First of all, let’s note that the system converges towards global satisfaction if $\tau = 0.375$; but, with $\tau = 0.625$, it always remains about 40% of unsatisfied agents in the population; so there is quasi-convergence only. In spite of the quasi-convergence phenomena, we can observe that a high value of $\tau$ leads in the long term to a low value around $-0.306$ for the $\text{segMix}$ index which reveals mixing: the satisfied agents are then organized as an alternation of homogeneous lines or columns constituted by agents with the same type (figure ??). As $\tau$ decreases, the $\text{segMix}$ index increases too and so mixing decreases (figure ??).

**Micro-Macro congruence** The gap between the threshold of tolerance and the mean utility over the whole population is surprisingly low at the end of a run (if $\tau = 0.375$, $mM = 0.73$ and if $\tau = 0.625$, $mM = 0.96$); in this way, complex dynamics build needed liveable configurations only.
Figure 4: $M_0$ model: attraction for the unlike or repulsion for the like.

$M_1^*$ model: repulsion for the like

In the $M_1^*$ model agents are satisfied if the normalized number of dissimilar neighbours is below the threshold. Because this model is the complement of $M_1^*$, micro-repulsion for similar neighbours will induces macro-mixing for low value of $\tau$.

Mixing  Simulations show that the system converges toward a population of satisfied agents where a low value around $-0.421$ for the segMix index reveals a strong mixing: the satisfied agents are mainly organized as an alternation of homogeneous lines or columns constituted by agents with the same type (figure ??). Let’s note that the vacant cells are placed at the crossroad between line and column and so allow right or left turns in the structure. As $\tau$ increases, the segMix index increases to a value close to zero and so mixing disappears (figure ??).

Micro-Macro congruence  There is high congruence between micro-motive and macro behaviour at the end of a run (if $\tau = 0.375$, $mM = 0.69$ and if $\tau = 0.625$, $mM = 0.66$); in this way, complex dynamics build needed liveable configurations almost only.

Figure 5: $M_1$ model: attraction for the unlike

(a) $\tau=0.375$ $a=-0.189$, $mM=0.73$
(b) $\tau=0.625$ $a=-0.306$, $m=0.96$ $s=60$

Figure 6: $M_1^*$ model: repulsion for the like.

(a) $\tau=0.375$ $a=-0.421$, $m=0.69$
(b) $\tau=0.625$ $a=-0.081$, $m=0.66$
Discussion and conclusion

Taking inspiration from the Schelling’s segregation model, we have proposed a family of models to study the effects of various micro-motives based on attraction or repulsion on the emergence of macro segregation or mixing. Table ?? summarizes results obtained from simulations; all the values are averaged over 100 independent runs.

The segregate-mixing index reveals: (i) three expected cases with strong segregation (bold values), (ii) three expected cases with strong mixing (bold values), (iii) two surprising cases with segregation: tolerant agents in the Schelling-$M_0$ model and unappealing agents in the $M^*_1$ model lead to macro-segregation (underline values), (iv) no unexpected case with mixing.

The micro-Macro index reveals: (i) for all the mixing models the micro-macro congruence is high (above 0.6) (ii) for all the segregation models the micro-macro congruence is low (below 0.5) (iii) two extreme cases (bold values): intolerant agents in the $M_0$ model lead to build much more liveable local configurations than necessary and appealing agents in the $M_1$ model lead to build needed liveable configurations only.

Measuring occupancy of frontier reveals that strong segregation comes with low occupancy (bold values).

Measuring porosity of frontier reveals: (i) high porosity emerges from tolerant or unappealing agents (bold values), (ii) the lower porosity comes with strong segregation.

We now attempt to provide elements to respond to the issues formulated in introduction.

Q1 Comparison between $M_0$ and $M_1$ (or $\overline{M}_0$ and $\overline{M}_1$) shows that it is not equivalent to flee regarding the proportion of dissimilar agents among the neighbourhood agents rather the number of dissimilar agents in the full set of neighbours.

Q2 (a) Because $M_0$ (resp. $\overline{M}_0$) is self-dual, it is equivalent to be attracted by similar (resp. dissimilar) or repulsed by dissimilar (resp. similar) if each one is influenced by the proportion of similar among its neighbours.

(b) Because $M_1$ and $M^*_1$ (or $\overline{M}_1$ and $\overline{M}^*_1$) are dual and the respective macro behaviours are different, it is not equivalent to be attracted by similar (resp. dissimilar) rather to be repulsed by dissimilar (resp. similar) neighbours if each one is influenced by the number of similar among its neighbours.

<table>
<thead>
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<th>$\tau$</th>
<th>segMix</th>
<th>mM</th>
<th>occup.</th>
<th>poro.</th>
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