Symmetries
Of Patterns in the Plane

**Translation** is a shift in a given direction.

... \[ \begin{array}{cccc} & \text{Pattern} \end{array} \] ...

An infinitely repeating pattern has translation as a symmetry - the entire pattern can be shifted over without changing in appearance:

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**Mirror Reflection** is a reflection across an imaginary line.

\[ \begin{array}{c} 
\text{Pattern} \\
\text{Pattern} \\
\end{array} \]

A pattern has mirror reflection as a symmetry if the entire pattern can reflect across a line yet remain unchanged:

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**Glide Reflection** composes translation and mirror reflection: A glide reflection reflects across a mirror line at the same time as translating along it.

\[ \begin{array}{c} 
\text{Pattern} \\
\text{Pattern} \\
\end{array} \]

Infinitely repeating patterns can have glide reflections:
Rotation is a symmetry that turns a pattern around a point.

$\frac{1}{2}$ turn rotation:

$\frac{1}{4}$ turn rotation:
$\frac{1}{3}$ turn rotation:

$\frac{1}{6}$ turn rotation:

These symmetries can be combined in patterns that repeat infinitely. Play with their combinations: [http://beautifulsymmetry.onl/wallpaper](http://beautifulsymmetry.onl/wallpaper).
This is the simplest symmetry group. It consists only of translations.

// CHALLENGES

Can you see both the horizontal translations and the vertical translations?

Can you see the diagonal translations?

// WITH COLOR

Can you color the pattern so that the shortest distance that the pattern can translate doubles?

// See the symmetries in animation: http://beautifulsymmetry.onl/wallpaper
// Symmetries quick guide: http://beautifulsymmetry.onl/symmetries
This group has glide reflections, and has neither rotations nor mirror reflections.

// CHALLENGES

Can you see the different parallel axes of the glide reflections?

// WITH COLOR

Can you color the pattern to remove the glide reflection so the only symmetry is translation?

// See the symmetries in animation: http://beautifulsymmetry.onl/wallpaper
// Symmetries quick guide: http://beautifulsymmetry.onl/symmetries
This group has $\frac{1}{2}$ turn rotations.

// CHALLENGES

Can you find all the different centers of the $\frac{1}{2}$ turn rotations?

// WITH COLOR

Could you color the pattern to remove half of the rotations?

Can you color the pattern to remove the rotations so that there is only translation?

// See the symmetries in animation: http://beautifulsymmetry.onl/wallpaper
// Symmetries quick guide: http://beautifulsymmetry.onl/symmetries
This group has \( \frac{1}{4} \) turn and \( \frac{1}{2} \) turn rotations. The centers of the \( \frac{1}{2} \) turn rotations are midway between the centers of the \( \frac{1}{4} \) turn rotations. It has no reflections.

// CHALLENGES
Can you see the different \( \frac{1}{4} \) turn rotations?

Can you see the different \( \frac{1}{2} \) turn rotations?

// WITH COLOR
Can you color the pattern so that it only has \( \frac{1}{2} \) turn rotations and translations and no \( \frac{1}{4} \) turn rotations?

Can you then add more color so that the pattern has no rotations, and only has translation?

// See the symmetries in animation: http://beautifulsymmetry.onl/wallpaper
// Symmetries quick guide: http://beautifulsymmetry.onl/symmetries
This group has perpendicular mirror reflections, and ½ turn rotations that lie both on and off the mirror reflection axes.

// CHALLENGES

Can you see the different perpendicular axes of mirror reflection and see where those axes intersect?

Can you see the ½ turn rotations that lie on the intersections of mirror reflection axes?

Can you see the ½ turn rotations that do not lie on the intersections of mirror reflection axes?

// WITH COLOR

Can you color the pattern to remove the vertical mirror reflections, but leave the horizontal mirror reflections?

Can you color the pattern to remove all mirror reflections?

// See the symmetries in animation: http://beautifulsymmetry.onl/wallpaper
// Symmetries quick guide: http://beautifulsymmetry.onl/symmetries
2 * 22
4*2

This group contains reflections as well as $\frac{1}{2}$ turn and $\frac{1}{4}$ turn rotations. There are two perpendicular axes of mirror reflection passing through each $\frac{1}{2}$ turn rotation, while there are four different glide reflections, with two glide reflection axes passing through each $\frac{1}{4}$ turn rotation.

// CHALLENGES

Can you see the $\frac{1}{2}$ turn and $\frac{1}{4}$ turn rotations?

Can you see the different mirror reflections?

Can you see the different glide reflections?

// WITH COLOR

Can you color the pattern to remove the $\frac{1}{4}$ turn rotations (but keep the $\frac{1}{2}$ turn rotations)?

Can you color the pattern to remove the vertical mirror reflections?

// See the symmetries in animation: http://beautifulsymmetry.onl/wallpaper
// Symmetries quick guide: http://beautifulsymmetry.onl/symmetries