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# **Introduction to Algorithms**

*Third Edition*

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# Index

This index uses the following conventions. Numbers are alphabetized as if spelled out; for example, “2-3-4 tree” is indexed as if it were “two-three-four tree.” When an entry refers to a place other than the main text, the page number is followed by a tag: ex. for exercise, pr. for problem, fig. for figure, and n. for footnote. A tagged page number often indicates the first page of an exercise or problem, which is not necessarily the page on which the reference actually appears.

- $\alpha(n)$ , 574
- $\phi$  (golden ratio), 59, 108 pr.
- $\hat{\phi}$  (conjugate of the golden ratio), 59
- $\phi(n)$  (Euler’s phi function), 943
- $\rho(n)$ -approximation algorithm, 1106, 1123
- $o$ -notation, 50–51, 64
- $O$ -notation, 45 fig., 47–48, 64
- $Q'$ -notation, 62 pr.
- $\tilde{O}$ -notation, 62 pr.
- $\omega$ -notation, 51
- $\Omega$ -notation, 45 fig., 48–49, 64
- $\tilde{\Omega}$ -notation, 62 pr.
- $\tilde{\Omega}$ -notation, 62 pr.
- $\Theta$ -notation, 44–47, 45 fig., 64
- $\tilde{\Theta}$ -notation, 62 pr.
- $\{ \}$  (set), 1158
- $\in$  (set member), 1158
- $\notin$  (not a set member), 1158
- $\emptyset$ 
  - (empty language), 1058
  - (empty set), 1158
- $\subseteq$  (subset), 1159
- $\subset$  (proper subset), 1159
- $:$  (such that), 1159
- $\cap$  (set intersection), 1159
- $\cup$  (set union), 1159
- $-$  (set difference), 1159
- $| |$ 
  - (flow value), 710
  - (length of a string), 986
  - (set cardinality), 1161
- $\times$ 
  - (Cartesian product), 1162
  - (cross product), 1016
- $\langle \rangle$ 
  - (sequence), 1166
  - (standard encoding), 1057
- $\binom{m}{k}$  (choose), 1185
- $\| \|$  (euclidean norm), 1222
- $!$  (factorial), 57
- $\lceil \rceil$  (ceiling), 54
- $\lfloor \rfloor$  (floor), 54
- $\sqrt{\phantom{x}}$  (lower square root), 546
- $\sqrt{\phantom{x}}$  (upper square root), 546
- $\sum$  (sum), 1145
- $\prod$  (product), 1148
- $\rightarrow$  (adjacency relation), 1169
- $\rightsquigarrow$  (reachability relation), 1170
- $\wedge$  (AND), 697, 1071
- $\neg$  (NOT), 1071
- $\vee$  (OR), 697, 1071
- $\oplus$  (group operator), 939
- $\otimes$  (convolution operator), 901

- \* (closure operator), 1058
- | (divides relation), 927
- ‡ (does-not-divide relation), 927
- $\equiv$  (equivalent modulo  $n$ ), 54, 1165 ex.
- $\not\equiv$  (not equivalent modulo  $n$ ), 54
- $[a]_n$  (equivalence class modulo  $n$ ), 928
- $+_n$  (addition modulo  $n$ ), 940
- $\cdot_n$  (multiplication modulo  $n$ ), 940
- $\left(\frac{a}{p}\right)$  (Legendre symbol), 982 pr.
- $\varepsilon$  (empty string), 986, 1058
- $\sqsubset$  (prefix relation), 986
- $\sqsupset$  (suffix relation), 986
- $\succ_x$  (above relation), 1022
- // (comment symbol), 21
- $\gg$  (much-greater-than relation), 574
- $\ll$  (much-less-than relation), 783
- $\leq_P$  (polynomial-time reducibility relation), 1067, 1077 ex.
  
- AA-tree, 338
- abelian group, 940
- ABOVE, 1024
- above relation ( $\succ_x$ ), 1022
- absent child, 1178
- absolutely convergent series, 1146
- absorption laws for sets, 1160
- abstract problem, 1054
- acceptable pair of integers, 972
- acceptance
  - by an algorithm, 1058
  - by a finite automaton, 996
- accepting state, 995
- accounting method, 456–459
  - for binary counters, 458
  - for dynamic tables, 465–466
  - for stack operations, 457–458, 458 ex.
- Ackermann's function, 585
- activity-selection problem, 415–422, 450
- acyclic graph, 1170
  - relation to matroids, 448 pr.
- add instruction, 23
- addition
  - of binary integers, 22 ex.
  - of matrices, 1220
  - modulo  $n$  ( $+_n$ ), 940
  - of polynomials, 898
- additive group modulo  $n$ , 940
- addressing, open, *see* open-address hash table
  
- ADD-SUBARRAY, 805 pr.
- adjacency-list representation, 590
  - replaced by a hash table, 593 ex.
- adjacency-matrix representation, 591
- adjacency relation ( $\rightarrow$ ), 1169
- adjacent vertices, 1169
- admissible edge, 749
- admissible network, 749–750
- adversary, 190
- aggregate analysis, 452–456
  - for binary counters, 454–455
  - for breadth-first search, 597
  - for depth-first search, 606
  - for Dijkstra's algorithm, 661
  - for disjoint-set data structures, 566–567, 568 ex.
  - for dynamic tables, 465
  - for Fibonacci heaps, 518, 522 ex.
  - for Graham's scan, 1036
  - for the Knuth-Morris-Pratt algorithm, 1006
  - for Prim's algorithm, 636
  - for rod-cutting, 367
  - for shortest paths in a dag, 655
  - for stack operations, 452–454
- aggregate flow, 863
- Akra-Bazzi method for solving a recurrence, 112–113
- algorithm, 5
  - correctness of, 6
  - origin of word, 42
  - running time of, 25
  - as a technology, 13
- Alice, 959
- ALLOCATE-NODE, 492
- ALLOCATE-OBJECT, 244
- allocation of objects, 243–244
- all-pairs shortest paths, 644, 684–707
  - in dynamic graphs, 707
  - in  $\epsilon$ -dense graphs, 706 pr.
  - Floyd-Warshall algorithm for, 693–697, 706
  - Johnson's algorithm for, 700–706
  - by matrix multiplication, 686–693, 706–707
  - by repeated squaring, 689–691
- alphabet, 995, 1057
- $\alpha(n)$ , 574
- amortized analysis, 451–478
  - accounting method of, 456–459
  - aggregate analysis, 367, 452–456

- for bit-reversal permutation, 472 pr.
- for breadth-first search, 597
- for depth-first search, 606
- for Dijkstra's algorithm, 661
- for disjoint-set data structures, 566–567, 568 ex., 572 ex., 575–581, 581–582 ex.
- for dynamic tables, 463–471
- for Fibonacci heaps, 509–512, 517–518, 520–522, 522 ex.
- for the generic push-relabel algorithm, 746
- for Graham's scan, 1036
- for the Knuth-Morris-Pratt algorithm, 1006
- for making binary search dynamic, 473 pr.
- potential method of, 459–463
- for restructuring red-black trees, 474 pr.
- for self-organizing lists with move-to-front, 476 pr.
- for shortest paths in a dag, 655
- for stacks on secondary storage, 502 pr.
- for weight-balanced trees, 473 pr.
- amortized cost
  - in the accounting method, 456
  - in aggregate analysis, 452
  - in the potential method, 459
- ancestor, 1176
  - least common, 584 pr.
- AND function ( $\wedge$ ), 697, 1071
- AND gate, 1070
- and, in pseudocode, 22
- antiparallel edges, 711–712
- antisymmetric relation, 1164
- ANY-SEGMENTS-INTERSECT, 1025
- approximation
  - by least squares, 835–839
  - of summation by integrals, 1154–1156
- approximation algorithm, 10, 1105–1140
  - for bin packing, 1134 pr.
  - for MAX-CNF satisfiability, 1127 ex.
  - for maximum clique, 1111 ex., 1134 pr.
  - for maximum matching, 1135 pr.
  - for maximum spanning tree, 1137 pr.
  - for maximum-weight cut, 1127 ex.
  - for MAX-3-CNF satisfiability, 1123–1124, 1139
  - for minimum-weight vertex cover, 1124–1127, 1139
  - for parallel machine scheduling, 1136 pr.
  - randomized, 1123
  - for set cover, 1117–1122, 1139
  - for subset sum, 1128–1134, 1139
  - for traveling-salesman problem, 1111–1117, 1139
  - for vertex cover, 1108–1111, 1139
  - for weighted set cover, 1135 pr.
  - for 0-1 knapsack problem, 1137 pr., 1139
- approximation error, 836
- approximation ratio, 1106, 1123
- approximation scheme, 1107
- APPROX-MIN-WEIGHT-VC, 1126
- APPROX-SUBSET-SUM, 1131
- APPROX-TSP-TOUR, 1112
- APPROX-VERTEX-COVER, 1109
- arbitrage, 679 pr.
- arc, *see* edge
- argument of a function, 1166–1167
- arithmetic instructions, 23
- arithmetic, modular, 54, 939–946
- arithmetic series, 1146
- arithmetic with infinities, 650
- arm, 485
- array, 21
  - Monge, 110 pr.
  - passing as a parameter, 21
- articulation point, 621 pr.
- assignment
  - multiple, 21
  - satisfying, 1072, 1079
  - truth, 1072, 1079
- associative laws for sets, 1160
- associative operation, 939
- asymptotically larger, 52
- asymptotically nonnegative, 45
- asymptotically positive, 45
- asymptotically smaller, 52
- asymptotically tight bound, 45
- asymptotic efficiency, 43
- asymptotic lower bound, 48
- asymptotic notation, 43–53, 62 pr.
  - and graph algorithms, 588
  - and linearity of summations, 1146
- asymptotic upper bound, 47
- attribute of an object, 21
- augmentation of a flow, 716
- augmenting data structures, 339–355
- augmenting path, 719–720, 763 pr.
- authentication, 284 pr., 960–961, 964

- automaton
  - finite, 995
  - string-matching, 996–1002
- auxiliary hash function, 272
- auxiliary linear program, 886
- average-case running time, 28, 116
- AVL-INSERT, 333 pr.
- AVL tree, 333 pr., 337
- axioms, for probability, 1190
  
- babyface, 602 ex.
- back edge, 609, 613
- back substitution, 817
- BAD-SET-COVER-INSTANCE, 1122 ex.
- BALANCE, 333 pr.
- balanced search tree
  - AA-trees, 338
  - AVL trees, 333 pr., 337
  - B-trees, 484–504
  - $k$ -neighbor trees, 338
  - red-black trees, 308–338
  - scapegoat trees, 338
  - splay trees, 338, 482
  - treaps, 333 pr., 338
  - 2-3-4 trees, 489, 503 pr.
  - 2-3 trees, 337, 504
  - weight-balanced trees, 338, 473 pr.
- balls and bins, 133–134, 1215 pr.
- base- $a$  pseudoprime, 967
- base case, 65, 84
- base, in DNA, 391
- basic feasible solution, 866
- basic solution, 866
- basic variable, 855
- basis function, 835
- Bayes's theorem, 1194
- BELLMAN-FORD, 651
- Bellman-Ford algorithm, 651–655, 682
  - for all-pairs shortest paths, 684
  - in Johnson's algorithm, 702–704
  - and objective functions, 670 ex.
  - to solve systems of difference constraints, 668
  - Yen's improvement to, 678 pr.
- BELOW, 1024
- Bernoulli trial, 1201
  - and balls and bins, 133–134
  - and streaks, 135–139
- best-case running time, 29 ex., 49
- BFS, 595
- BIASED-RANDOM, 117 ex.
- biconnected component, 621 pr.
- big-oh notation, 45 fig., 47–48, 64
- big-omega notation, 45 fig., 48–49, 64
- bijective function, 1167
- binary character code, 428
- binary counter
  - analyzed by accounting method, 458
  - analyzed by aggregate analysis, 454–455
  - analyzed by potential method, 461–462
  - bit-reversed, 472 pr.
- binary entropy function, 1187
- binary gcd algorithm, 981 pr.
- binary heap, *see* heap
- binary relation, 1163
- binary search, 39 ex.
  - with fast insertion, 473 pr.
  - in insertion sort, 39 ex.
  - in multithreaded merging, 799–800
  - in searching B-trees, 499 ex.
- BINARY-SEARCH, 799
- binary search tree, 286–307
  - AA-trees, 338
  - AVL trees, 333 pr., 337
  - deletion from, 295–298, 299 ex.
  - with equal keys, 303 pr.
  - insertion into, 294–295
  - $k$ -neighbor trees, 338
  - maximum key of, 291
  - minimum key of, 291
  - optimal, 397–404, 413
  - predecessor in, 291–292
  - querying, 289–294
  - randomly built, 299–303, 304 pr.
  - right-converting of, 314 ex.
  - scapegoat trees, 338
  - searching, 289–291
  - for sorting, 299 ex.
  - splay trees, 338
  - successor in, 291–292
  - and treaps, 333 pr.
  - weight-balanced trees, 338
  - see also* red-black tree
- binary-search-tree property, 287
  - in treaps, 333 pr.
  - vs. min-heap property, 289 ex.

- binary tree, 1177
  - full, 1178
  - number of different ones, 306 pr.
  - representation of, 246
  - superimposed upon a bit vector, 533–534
  - see also* binary search tree
- binomial coefficient, 1186–1187
- binomial distribution, 1203–1206
  - and balls and bins, 133
  - maximum value of, 1207 ex.
  - tails of, 1208–1215
- binomial expansion, 1186
- binomial heap, 527 pr.
- binomial tree, 527 pr.
- bin packing, 1134 pr.
- bipartite graph, 1172
  - corresponding flow network of, 732
  - $d$ -regular, 736 ex.
  - and hypergraphs, 1173 ex.
- bipartite matching, 530, 732–736, 747 ex., 766
  - Hopcroft-Karp algorithm for, 763 pr.
- birthday paradox, 130–133, 142 ex.
- bisection of a tree, 1181 pr.
- bitonic euclidean traveling-salesman problem, 405 pr.
- bitonic sequence, 682 pr.
- bitonic tour, 405 pr.
- bit operation, 927
  - in Euclid's algorithm, 981 pr.
- bit-reversal permutation, 472 pr., 918
- BIT-REVERSE-COPY, 918
- bit-reversed binary counter, 472 pr.
- BIT-REVERSED-INCREMENT, 472 pr.
- bit vector, 255 ex., 532–536
- black-height, 309
- black vertex, 594, 603
- blocking flow, 765
- block structure in pseudocode, 20
- Bob, 959
- Boole's inequality, 1195 ex.
- boolean combinational circuit, 1071
- boolean combinational element, 1070
- boolean connective, 1079
- boolean formula, 1049, 1066 ex., 1079, 1086 ex.
- boolean function, 1187 ex.
- boolean matrix multiplication, 832 ex.
- Borůvka's algorithm, 641
- bottleneck spanning tree, 640 pr.
- bottleneck traveling-salesman problem, 1117 ex.
- bottom of a stack, 233
- BOTTOM-UP-CUT-ROD, 366
- bottom-up method, for dynamic programming, 365
- bound
  - asymptotically tight, 45
  - asymptotic lower, 48
  - asymptotic upper, 47
  - on binomial coefficients, 1186–1187
  - on binomial distributions, 1206
  - polylogarithmic, 57
  - on the tails of a binomial distribution, 1208–1215
  - see also* lower bounds
- boundary condition, in a recurrence, 67, 84
- boundary of a polygon, 1020 ex.
- bounding a summation, 1149–1156
- box, nesting, 678 pr.
- $B^+$ -tree, 488
- branching factor, in B-trees, 487
- branch instructions, 23
- breadth-first search, 594–602, 623
  - in maximum flow, 727–730, 766
  - and shortest paths, 597–600, 644
  - similarity to Dijkstra's algorithm, 662, 663 ex.
- breadth-first tree, 594, 600
- bridge, 621 pr.
- $B^*$ -tree, 489 n.
- B-tree, 484–504
  - compared with red-black trees, 484, 490
  - creating, 492
  - deletion from, 499–502
  - full node in, 489
  - height of, 489–490
  - insertion into, 493–497
  - minimum degree of, 489
  - minimum key of, 497 ex.
  - properties of, 488–491
  - searching, 491–492
  - splitting a node in, 493–495
  - 2-3-4 trees, 489
- B-TREE-CREATE, 492
- B-TREE-DELETE, 499
- B-TREE-INSERT, 495

- B-TREE-INSERT-NONFULL, 496
- B-TREE-SEARCH, 492, 499 ex.
- B-TREE-SPLIT-CHILD, 494
- BUBBLESORT, 40 pr.
- bucket, 200
- bucket sort, 200–204
- BUCKET-SORT, 201
- BUILD-MAX-HEAP, 157
- BUILD-MAX-HEAP', 167 pr.
- BUILD-MIN-HEAP, 159
- butterfly operation, 915
- by**, in pseudocode, 21
  
- cache, 24, 449 pr.
- cache hit, 449 pr.
- cache miss, 449 pr.
- cache obliviousness, 504
- caching, off-line, 449 pr.
- call
  - in a multithreaded computation, 776
  - of a subroutine, 23, 25 n.
  - by value, 21
- call edge, 778
- cancellation lemma, 907
- cancellation of flow, 717
- canonical form for task scheduling, 444
- capacity
  - of a cut, 721
  - of an edge, 709
  - residual, 716, 719
  - of a vertex, 714 ex.
- capacity constraint, 709–710
- cardinality of a set ( $| \cdot |$ ), 1161
- Carmichael number, 968, 975 ex.
- Cartesian product ( $\times$ ), 1162
- Cartesian sum, 906 ex.
- cascading cut, 520
- CASCADING-CUT, 519
- Catalan numbers, 306 pr., 372
- ceiling function ( $\lceil \cdot \rceil$ ), 54
  - in master theorem, 103–106
- ceiling instruction, 23
- certain event, 1190
- certificate
  - in a cryptosystem, 964
  - for verification algorithms, 1063
- CHAINED-HASH-DELETE, 258
- CHAINED-HASH-INSERT, 258
- CHAINED-HASH-SEARCH, 258
- chaining, 257–260, 283 pr.
- chain of a convex hull, 1038
- changing a key, in a Fibonacci heap, 529 pr.
- changing variables, in the substitution method, 86–87
- character code, 428
- chess-playing program, 790–791
- child
  - in a binary tree, 1178
  - in a multithreaded computation, 776
  - in a rooted tree, 1176
- child list in a Fibonacci heap, 507
- Chinese remainder theorem, 950–954, 983
- chip multiprocessor, 772
- chirp transform, 914 ex.
- choose  $\binom{n}{k}$ , 1185
- chord, 345 ex.
- Cilk, 774, 812
- Cilk++, 774, 812
- ciphertext, 960
- circuit
  - boolean combinational, 1071
  - depth of, 919
  - for fast Fourier transform, 919–920
- CIRCUIT-SAT, 1072
- circuit satisfiability, 1070–1077
- circular, doubly linked list with a sentinel, 239
- circular linked list, 236
  - see also* linked list
- class
  - complexity, 1059
  - equivalence, 1164
- classification of edges
  - in breadth-first search, 621 pr.
  - in depth-first search, 609–610, 611 ex.
  - in a multithreaded dag, 778–779
- clause, 1081–1082
- clean area, 208 pr.
- clique, 1086–1089, 1105
  - approximation algorithm for, 1111 ex., 1134 pr.
- CLIQUE, 1087
- closed interval, 348
- closed semiring, 707
- closest pair, finding, 1039–1044, 1047
- closest-point heuristic, 1117 ex.

- closure
  - group property, 939
  - of a language, 1058
  - operator (\*), 1058
  - transitive, *see* transitive closure
- cluster
  - in a bit vector with a superimposed tree of constant height, 534
  - for parallel computing, 772
  - in proto van Emde Boas structures, 538
  - in van Emde Boas trees, 546
- clustering, 272
- CNF (conjunctive normal form), 1049, 1082
- CNF satisfiability, 1127 *ex.*
- coarsening leaves of recursion
  - in merge sort, 39 *pr.*
  - when recursively spawning, 787
- code, 428–429
  - Huffman, 428–437, 450
- codeword, 429
- codomain, 1166
- coefficient
  - binomial, 1186
  - of a polynomial, 55, 898
  - in slack form, 856
- coefficient representation, 900
  - and fast multiplication, 903–905
- cofactor, 1224
- coin changing, 446 *pr.*
- colinearity, 1016
- collision, 257
  - resolution by chaining, 257–260
  - resolution by open addressing, 269–277
- collision-resistant hash function, 964
- coloring, 1103 *pr.*, 1180 *pr.*
- color, of a red-black-tree node, 308
- column-major order, 208 *pr.*
- column rank, 1223
- columnsort, 208 *pr.*
- column vector, 1218
- combination, 1185
- combinational circuit, 1071
- combinational element, 1070
- combine step, in divide-and-conquer, 30, 65
- comment, in pseudocode (//), 21
- commodity, 862
- common divisor, 929
  - greatest, *see* greatest common divisor
- common multiple, 939 *ex.*
- common subexpression, 915
- common subsequence, 7, 391
  - longest, 7, 390–397, 413
- commutative laws for sets, 1159
- commutative operation, 940
- COMPACTIFY-LIST, 245 *ex.*
- compact list, 250 *pr.*
- COMPACT-LIST-SEARCH, 250 *pr.*
- COMPACT-LIST-SEARCH', 251 *pr.*
- comparable line segments, 1022
- COMPARE-EXCHANGE, 208 *pr.*
- compare-exchange operation, 208 *pr.*
- comparison sort, 191
  - and binary search trees, 289 *ex.*
  - randomized, 205 *pr.*
  - and selection, 222
- compatible activities, 415
- compatible matrices, 371, 1221
- competitive analysis, 476 *pr.*
- complement
  - of an event, 1190
  - of a graph, 1090
  - of a language, 1058
  - Schur, 820, 834
  - of a set, 1160
- complementary slackness, 894 *pr.*
- complete graph, 1172
- complete  $k$ -ary tree, 1179
  - see also* heap
- completeness of a language, 1077 *ex.*
- complete step, 782
- completion time, 447 *pr.*, 1136 *pr.*
- complexity class, 1059
  - co-NP, 1064
  - NP, 1049, 1064
  - NPC, 1050, 1069
  - P, 1049, 1055
- complexity measure, 1059
- complex numbers
  - inverting matrices of, 832 *ex.*
  - multiplication of, 83 *ex.*
- complex root of unity, 906
  - interpolation at, 912–913
- component
  - biconnected, 621 *pr.*
  - connected, 1170
  - strongly connected, 1170

- component graph, 617
- composite number, 928
  - witness to, 968
- composition, of multithreaded computations, 784 fig.
- computational depth, 812
- computational geometry, 1014–1047
- computational problem, 5–6
- computation dag, 777
- computation, multithreaded, 777
- COMPUTE-PREFIX-FUNCTION, 1006
- COMPUTE-TRANSITION-FUNCTION, 1001
- concatenation
  - of languages, 1058
  - of strings, 986
- concrete problem, 1055
- concurrency keywords, 774, 776, 785
- concurrency platform, 773
- conditional branch instruction, 23
- conditional independence, 1195 ex.
- conditional probability, 1192, 1194
- configuration, 1074
- conjugate of the golden ratio ( $\hat{\phi}$ ), 59
- conjugate transpose, 832 ex.
- conjunctive normal form, 1049, 1082
- connected component, 1170
  - identified using depth-first search, 612 ex.
  - identified using disjoint-set data structures, 562–564
- CONNECTED-COMPONENTS, 563
- connected graph, 1170
- connective, 1079
- co-NP (complexity class), 1064
- conquer step, in divide-and-conquer, 30, 65
- conservation of flow, 709–710
- consistency
  - of literals, 1088
  - sequential, 779, 812
- CONSOLIDATE, 516
- consolidating a Fibonacci-heap root list, 513–517
- constraint, 851
  - difference, 665
  - equality, 670 ex., 852–853
  - inequality, 852–853
  - linear, 846
  - nonnegativity, 851, 853
  - tight, 865
  - violation of, 865
- constraint graph, 666–668
- contain, in a path, 1170
- continuation edge, 778
- continuous uniform probability distribution, 1192
- contraction
  - of a dynamic table, 467–471
  - of a matroid, 442
  - of an undirected graph by an edge, 1172
- control instructions, 23
- convergence property, 650, 672–673
- convergent series, 1146
- converting binary to decimal, 933 ex.
- convex combination of points, 1015
- convex function, 1199
- convex hull, 8, 1029–1039, 1046 pr.
- convex layers, 1044 pr.
- convex polygon, 1020 ex.
- convex set, 714 ex.
- convolution ( $\otimes$ ), 901
- convolution theorem, 913
- copy instruction, 23
- correctness of an algorithm, 6
- corresponding flow network for bipartite matching, 732
- countably infinite set, 1161
- counter, *see* binary counter
- counting, 1183–1189
  - probabilistic, 143 pr.
- counting sort, 194–197
  - in radix sort, 198
- COUNTING-SORT, 195
- coupon collector's problem, 134
- cover
  - path, 761 pr.
  - by a subset, 1118
  - vertex, 1089, 1108, 1124–1127, 1139
- coververtical, 1024
- CREATE-NEW-RS-V-EB-TREE, 557 pr.
- credit, 456
- critical edge, 729
- critical path
  - of a dag, 657
  - of a multithreaded computation, 779
- cross a cut, 626
- cross edge, 609
- cross product ( $\times$ ), 1016

- cryptosystem, 958–965, 983
- cubic spline, 840 pr.
- currency exchange, 390 ex., 679 pr.
- curve fitting, 835–839
- cut
  - capacity of, 721
  - cascading, 520
  - of a flow network, 720–724
  - minimum, 721, 731 ex.
  - net flow across, 720
  - of an undirected graph, 626
  - weight of, 1127 ex.
- CUT, 519
- CUT-ROD, 363
- cutting, in a Fibonacci heap, 519
- cycle of a graph, 1170
  - hamiltonian, 1049, 1061
  - minimum mean-weight, 680 pr.
  - negative-weight, *see* negative-weight cycle and shortest paths, 646–647
- cyclic group, 955
- cyclic rotation, 1012 ex.
- cycling, of simplex algorithm, 875
  
- dag, *see* directed acyclic graph
- DAG-SHORTEST-PATHS, 655
- $d$ -ary heap, 167 pr.
  - in shortest-paths algorithms, 706 pr.
- data-movement instructions, 23
- data-parallel model, 811
- data structure, 9, 229–355, 481–585
  - AA-trees, 338
  - augmentation of, 339–355
  - AVL trees, 333 pr., 337
  - binary search trees, 286–307
  - binomial heaps, 527 pr.
  - bit vectors, 255 ex., 532–536
  - B-trees, 484–504
  - deques, 236 ex.
  - dictionaries, 229
  - direct-address tables, 254–255
  - for disjoint sets, 561–585
  - for dynamic graphs, 483
  - dynamic sets, 229–231
  - dynamic trees, 482
  - exponential search trees, 212, 483
  - Fibonacci heaps, 505–530
  - fusion trees, 212, 483
  - hash tables, 256–261
  - heaps, 151–169
  - interval trees, 348–354
  - $k$ -neighbor trees, 338
  - linked lists, 236–241
  - mergeable heap, 505
  - order-statistic trees, 339–345
  - persistent, 331 pr., 482
  - potential of, 459
  - priority queues, 162–166
  - proto van Emde Boas structures, 538–545
  - queues, 232, 234–235
  - radix trees, 304 pr.
  - red-black trees, 308–338
  - relaxed heaps, 530
  - rooted trees, 246–249
  - scapegoat trees, 338
  - on secondary storage, 484–487
  - skip lists, 338
  - splay trees, 338, 482
  - stacks, 232–233
  - treaps, 333 pr., 338
  - 2-3-4 heaps, 529 pr.
  - 2-3-4 trees, 489, 503 pr.
  - 2-3 trees, 337, 504
  - van Emde Boas trees, 531–560
  - weight-balanced trees, 338
- data type, 23
- deadline, 444
- deallocation of objects, 243–244
- decision by an algorithm, 1058–1059
- decision problem, 1051, 1054
  - and optimization problems, 1051
- decision tree, 192–193
- DECREASE-KEY, 162, 505
- decreasing a key
  - in Fibonacci heaps, 519–522
  - in 2-3-4 heaps, 529 pr.
- DECREMENT, 456 ex.
- degeneracy, 874
- degree
  - of a binomial-tree root, 527 pr.
  - maximum, of a Fibonacci heap, 509, 523–526
  - minimum, of a B-tree, 489
  - of a node, 1177
  - of a polynomial, 55, 898
  - of a vertex, 1169

- degree-bound, 898
- DELETE, 230, 505
- DELETE-LARGER-HALF, 463 ex.
- deletion
  - from binary search trees, 295–298, 299 ex.
  - from a bit vector with a superimposed binary tree, 534
  - from a bit vector with a superimposed tree of constant height, 535
  - from B-trees, 499–502
  - from chained hash tables, 258
  - from direct-address tables, 254
  - from dynamic tables, 467–471
  - from Fibonacci heaps, 522, 526 pr.
  - from heaps, 166 ex.
  - from interval trees, 349
  - from linked lists, 238
  - from open-address hash tables, 271
  - from order-statistic trees, 343–344
  - from proto van Emde Boas structures, 544
  - from queues, 234
  - from red-black trees, 323–330
  - from stacks, 232
  - from sweep-line statuses, 1024
  - from 2-3-4 heaps, 529 pr.
  - from van Emde Boas trees, 554–556
- DeMorgan's laws
  - for propositional logic, 1083
  - for sets, 1160, 1162 ex.
- dense graph, 589
- $\epsilon$ -dense, 706 pr.
- density
  - of prime numbers, 965–966
  - of a rod, 370 ex.
- dependence
  - and indicator random variables, 119
  - linear, 1223
  - see also* independence
- depth
  - average, of a node in a randomly built binary search tree, 304 pr.
  - of a circuit, 919
  - of a node in a rooted tree, 1177
  - of quicksort recursion tree, 178 ex.
  - of a stack, 188 pr.
- depth-determination problem, 583 pr.
- depth-first forest, 603
- depth-first search, 603–612, 623
  - in finding articulation points, bridges, and biconnected components, 621 pr.
  - in finding strongly connected components, 615–621, 623
  - in topological sorting, 612–615
- depth-first tree, 603
- deque, 236 ex.
- DEQUEUE, 235
- derivative of a series, 1147
- descendant, 1176
- destination vertex, 644
- det, *see* determinant
- determinacy race, 788
- determinant, 1224–1225
  - and matrix multiplication, 832 ex.
- deterministic algorithm, 123
  - multithreaded, 787
- DETERMINISTIC-SEARCH, 143 pr.
- DFS, 604
- DFS-VISIT, 604
- DFT (discrete Fourier transform), 9, 909
- diagonal matrix, 1218
  - LUP decomposition of, 827 ex.
- diameter of a tree, 602 ex.
- dictionary, 229
- difference constraints, 664–670
- difference equation, *see* recurrence
- difference of sets ( $-$ ), 1159
  - symmetric, 763 pr.
- differentiation of a series, 1147
- digital signature, 960
- digraph, *see* directed graph
- DIJKSTRA, 658
- Dijkstra's algorithm, 658–664, 682
  - for all-pairs shortest paths, 684, 704
  - implemented with a Fibonacci heap, 662
  - implemented with a min-heap, 662
  - with integer edge weights, 664 ex.
  - in Johnson's algorithm, 702
  - similarity to breadth-first search, 662, 663 ex.
  - similarity to Prim's algorithm, 634, 662
- DIRECT-ADDRESS-DELETE, 254
- direct addressing, 254–255, 532–536
- DIRECT-ADDRESS-INSERT, 254
- DIRECT-ADDRESS-SEARCH, 254
- direct-address table, 254–255
- directed acyclic graph (dag), 1172

- and back edges, 613
- and component graphs, 617
- and hamiltonian paths, 1066 ex.
- longest simple path in, 404 pr.
- for representing a multithreaded
  - computation, 777
- single-source shortest-paths algorithm for, 655–658
- topological sort of, 612–615, 623
- directed graph, 1168
  - all-pairs shortest paths in, 684–707
  - constraint graph, 666
  - Euler tour of, 623 pr., 1048
  - hamiltonian cycle of, 1049
  - and longest paths, 1048
  - path cover of, 761 pr.
  - PERT chart, 657, 657 ex.
  - semiconnected, 621 ex.
  - shortest path in, 643
  - single-source shortest paths in, 643–683
  - singly connected, 612 ex.
  - square of, 593 ex.
  - transitive closure of, 697
  - transpose of, 592 ex.
  - universal sink in, 593 ex.
  - see also* directed acyclic graph, graph, network
- directed segment, 1015–1017
- directed version of an undirected graph, 1172
- DIRECTION, 1018
- dirty area, 208 pr.
- DISCHARGE, 751
- discharge of an overflowing vertex, 751
- discovered vertex, 594, 603
- discovery time, in depth-first search, 605
- discrete Fourier transform, 9, 909
- discrete logarithm, 955
- discrete logarithm theorem, 955
- discrete probability distribution, 1191
- discrete random variable, 1196–1201
- disjoint-set data structure, 561–585
  - analysis of, 575–581, 581 ex.
  - in connected components, 562–564
  - in depth determination, 583 pr.
  - disjoint-set-forest implementation of, 568–572
  - in Kruskal’s algorithm, 631
  - linear-time special case of, 585
  - linked-list implementation of, 564–568
  - in off-line least common ancestors, 584 pr.
  - in off-line minimum, 582 pr.
  - in task scheduling, 448 pr.
- disjoint-set forest, 568–572
  - analysis of, 575–581, 581 ex.
  - rank properties of, 575, 581 ex.
  - see also* disjoint-set data structure
- disjoint sets, 1161
- disjunctive normal form, 1083
- disk, 1028 ex.
- disk drive, 485–487
  - see also* secondary storage
- DISK-READ, 487
- DISK-WRITE, 487
- distance
  - edit, 406 pr.
  - euclidean, 1039
  - $L_m$ , 1044 ex.
  - Manhattan, 225 pr., 1044 ex.
  - of a shortest path, 597
- distributed memory, 772
- distribution
  - binomial, 1203–1206
  - continuous uniform, 1192
  - discrete, 1191
  - geometric, 1202–1203
  - of inputs, 116, 122
  - of prime numbers, 965
  - probability, 1190
  - sparse-hulled, 1046 pr.
  - uniform, 1191
- distributive laws for sets, 1160
- divergent series, 1146
- divide-and-conquer method, 30–35, 65
  - analysis of, 34–35
  - for binary search, 39 ex.
  - for conversion of binary to decimal, 933 ex.
  - for fast Fourier transform, 909–912
  - for finding the closest pair of points, 1040–1043
  - for finding the convex hull, 1030
  - for matrix inversion, 829–831
  - for matrix multiplication, 76–83, 792–797
  - for maximum-subarray problem, 68–75
  - for merge sort, 30–37, 797–805
  - for multiplication, 920 pr.

- for multithreaded matrix multiplication, 792–797
  - for multithreaded merge sort, 797–805
  - for quicksort, 170–190
  - relation to dynamic programming, 359
  - for selection, 215–224
  - solving recurrences for, 83–106, 112–113
  - for Strassen’s algorithm, 79–83
- divide instruction, 23
- divides relation ( $\mid$ ), 927
- divide step, in divide-and-conquer, 30, 65
- division method, 263, 268–269 ex.
- division theorem, 928
- divisor, 927–928
  - common, 929
  - see also* greatest common divisor
- DNA, 6–7, 390–391, 406 pr.
- DNF (disjunctive normal form), 1083
- does-not-divide relation ( $\nmid$ ), 927
- domain, 1166
- dominates relation, 1045 pr.
- double hashing, 272–274, 277 ex.
- doubly linked list, 236
  - see also* linked list
- down to**, in pseudocode, 21
- $d$ -regular graph, 736 ex.
- duality, 879–886, 895 pr.
  - weak, 880–881, 886 ex.
- dual linear program, 879
- dummy key, 397
- dynamic graph, 562 n.
  - all-pairs shortest paths algorithms for, 707
  - data structures for, 483
  - minimum-spanning-tree algorithm for, 637 ex.
  - transitive closure of, 705 pr., 707
- dynamic multithreaded algorithm, *see* multithreaded algorithm
- dynamic multithreading, 773
- dynamic order statistics, 339–345
- dynamic-programming method, 359–413
  - for activity selection, 421 ex.
  - for all-pairs shortest paths, 686–697
  - for bitonic euclidean traveling-salesman problem, 405 pr.
  - bottom-up, 365
  - for breaking a string, 410 pr.
  - compared with greedy algorithms, 381, 390 ex., 418, 423–427
  - for edit distance, 406 pr.
  - elements of, 378–390
  - for Floyd-Warshall algorithm, 693–697
  - for inventory planning, 411 pr.
  - for longest common subsequence, 390–397
  - for longest palindrome subsequence, 405 pr.
  - for longest simple path in a weighted directed acyclic graph, 404 pr.
  - for matrix-chain multiplication, 370–378
  - and memoization, 387–389
  - for optimal binary search trees, 397–404
  - optimal substructure in, 379–384
  - overlapping subproblems in, 384–386
  - for printing neatly, 405 pr.
  - reconstructing an optimal solution in, 387
  - relation to divide-and-conquer, 359
  - for rod-cutting, 360–370
  - for seam carving, 409 pr.
  - for signing free agents, 411 pr.
  - top-down with memoization, 365
  - for transitive closure, 697–699
  - for Viterbi algorithm, 408 pr.
  - for 0-1 knapsack problem, 427 ex.
- dynamic set, 229–231
  - see also* data structure
- dynamic table, 463–471
  - analyzed by accounting method, 465–466
  - analyzed by aggregate analysis, 465
  - analyzed by potential method, 466–471
  - load factor of, 463
- dynamic tree, 482
- $e$ , 55
- E [] (expected value), 1197
- early-first form, 444
- early task, 444
- edge, 1168
  - admissible, 749
  - antiparallel, 711–712
  - attributes of, 592
  - back, 609
  - bridge, 621 pr.
  - call, 778
  - capacity of, 709
  - classification in breadth-first search, 621 pr.
  - classification in depth-first search, 609–610

- continuation, 778
- critical, 729
- cross, 609
- forward, 609
- inadmissible, 749
- light, 626
- negative-weight, 645–646
- residual, 716
- return, 779
- safe, 626
- saturated, 739
- spawn, 778
- tree, 601, 603, 609
- weight of, 591
- edge connectivity, 731 ex.
- edge set, 1168
- edit distance, 406 pr.
- Edmonds-Karp algorithm, 727–730
- elementary event, 1189
- elementary insertion, 465
- element of a set ( $\in$ ), 1158
- ellipsoid algorithm, 850, 897
- elliptic-curve factorization method, 984
- elseif**, in pseudocode, 20 n.
- else**, in pseudocode, 20
- empty language ( $\emptyset$ ), 1058
- empty set ( $\emptyset$ ), 1158
- empty set laws, 1159
- empty stack, 233
- empty string ( $\epsilon$ ), 986, 1058
- empty tree, 1178
- encoding of problem instances, 1055–1057
- endpoint
  - of an interval, 348
  - of a line segment, 1015
- ENQUEUE, 235
- entering a vertex, 1169
- entering variable, 867
- entropy function, 1187
- $\epsilon$ -dense graph, 706 pr.
- $\epsilon$ -universal hash function, 269 ex.
- equality
  - of functions, 1166
  - linear, 845
  - of sets, 1158
- equality constraint, 670 ex., 852
  - and inequality constraints, 853
  - tight, 865
  - violation of, 865
- equation
  - and asymptotic notation, 49–50
  - normal, 837
  - recurrence, *see* recurrence
- equivalence class, 1164
  - modulo  $n$  ( $[a]_n$ ), 928
- equivalence, modular ( $\equiv$ ), 54, 1165 ex.
- equivalence relation, 1164
  - and modular equivalence, 1165 ex.
- equivalent linear programs, 852
- error**, in pseudocode, 22
- escape problem, 760 pr.
- EUCLID, 935
- Euclid’s algorithm, 933–939, 981 pr., 983
- euclidean distance, 1039
- euclidean norm ( $\| \cdot \|$ ), 1222
- Euler’s constant, 943
- Euler’s phi function, 943
- Euler’s theorem, 954, 975 ex.
- Euler tour, 623 pr., 1048
  - and hamiltonian cycles, 1048
- evaluation of a polynomial, 41 pr., 900, 905 ex.
  - derivatives of, 922 pr.
  - at multiple points, 923 pr.
- event, 1190
- event point, 1023
- event-point schedule, 1023
- EXACT-SUBSET-SUM, 1129
- excess flow, 736
- exchange property, 437
- exclusion and inclusion, 1163 ex.
- execute a subroutine, 25 n.
- expansion of a dynamic table, 464–467
- expectation, *see* expected value
- expected running time, 28, 117
- expected value, 1197–1199
  - of a binomial distribution, 1204
  - of a geometric distribution, 1202
  - of an indicator random variable, 118
- explored vertex, 605
- exponential function, 55–56
- exponential height, 300
- exponential search tree, 212, 483
- exponential series, 1147
- exponentiation instruction, 24
- exponentiation, modular, 956
- EXTENDED-BOTTOM-UP-CUT-ROD, 369

- EXTENDED-EUCLID, 937
- EXTEND-SHORTEST-PATHS, 688
- extension of a set, 438
- exterior of a polygon, 1020 ex.
- external node, 1176
- external path length, 1180 ex.
- extracting the maximum key
  - from  $d$ -ary heaps, 167 pr.
  - from max-heaps, 163
- extracting the minimum key
  - from Fibonacci heaps, 512–518
  - from 2-3-4 heaps, 529 pr.
  - from Young tableaux, 167 pr.
- EXTRACT-MAX, 162–163
- EXTRACT-MIN, 162, 505
  
- factor, 928
  - twiddle, 912
- factorial function (!), 57–58
- factorization, 975–980, 984
  - unique, 931
- failure, in a Bernoulli trial, 1201
- fair coin, 1191
- fan-out, 1071
- Farkas's lemma, 895 pr.
- farthest-pair problem, 1030
- FASTER-ALL-PAIRS-SHORTEST-PATHS, 691, 692 ex.
- fast Fourier transform (FFT), 898–925
  - circuit for, 919–920
  - iterative implementation of, 915–918
  - multidimensional, 921 pr.
  - multithreaded algorithm for, 804 ex.
  - recursive implementation of, 909–912
  - using modular arithmetic, 923 pr.
- feasibility problem, 665, 894 pr.
- feasible linear program, 851
- feasible region, 847
- feasible solution, 665, 846, 851
- Fermat's theorem, 954
- FFT, *see* fast Fourier transform
- FFTW, 924
- FIB, 775
- FIB-HEAP-CHANGE-KEY, 529 pr.
- FIB-HEAP-DECREASE-KEY, 519
- FIB-HEAP-DELETE, 522
- FIB-HEAP-EXTRACT-MIN, 513
- FIB-HEAP-INSERT, 510
- FIB-HEAP-LINK, 516
- FIB-HEAP-PRUNE, 529 pr.
- FIB-HEAP-UNION, 512
- Fibonacci heap, 505–530
  - changing a key in, 529 pr.
  - compared with binary heaps, 506–507
  - creating, 510
  - decreasing a key in, 519–522
  - deletion from, 522, 526 pr.
  - in Dijkstra's algorithm, 662
  - extracting the minimum key from, 512–518
  - insertion into, 510–511
  - in Johnson's algorithm, 704
  - maximum degree of, 509, 523–526
  - minimum key of, 511
  - potential function for, 509
  - in Prim's algorithm, 636
  - pruning, 529 pr.
  - running times of operations on, 506 fig.
  - uniting, 511–512
- Fibonacci numbers, 59–60, 108 pr., 523
  - computation of, 774–780, 981 pr.
- FIFO (first-in, first-out), 232
  - see also* queue
- final-state function, 996
- final strand, 779
- FIND-DEPTH, 583 pr.
- FIND-MAX-CROSSING-SUBARRAY, 71
- FIND-MAXIMUM-SUBARRAY, 72
- find path, 569
- FIND-SET, 562
  - disjoint-set-forest implementation of, 571, 585
  - linked-list implementation of, 564
- finished vertex, 603
- finishing time, in depth-first search, 605
  - and strongly connected components, 618
- finish time, in activity selection, 415
- finite automaton, 995
  - for string matching, 996–1002
- FINITE-AUTOMATON-MATCHER, 999
- finite group, 940
- finite sequence, 1166
- finite set, 1161
- first-fit heuristic, 1134 pr.
- first-in, first-out, 232
  - see also* queue
- fixed-length code, 429

- floating-point data type, 23
- floor function ( $\lfloor \cdot \rfloor$ ), 54
  - in master theorem, 103–106
- floor instruction, 23
- flow, 709–714
  - aggregate, 863
  - augmentation of, 716
  - blocking, 765
  - cancellation of, 717
  - excess, 736
  - integer-valued, 733
  - net, across a cut, 720
  - value of, 710
- flow conservation, 709–710
- flow network, 709–714
  - corresponding to a bipartite graph, 732
  - cut of, 720–724
  - with multiple sources and sinks, 712
- FLOYD-WARSHALL, 695
- FLOYD-WARSHALL', 699 ex.
- Floyd-Warshall algorithm, 693–697, 699–700 ex., 706
  - multithreaded, 797 ex.
- FORD-FULKERSON, 724
- Ford-Fulkerson method, 714–731, 765
- FORD-FULKERSON-METHOD, 715
- forest, 1172–1173
  - depth-first, 603
  - disjoint-set, 568–572
- for**, in pseudocode, 20–21
  - and loop invariants, 19 n.
- formal power series, 108 pr.
- formula satisfiability, 1079–1081, 1105
- forward edge, 609
- forward substitution, 816–817
- Fourier transform, *see* discrete Fourier transform, fast Fourier transform
- fractional knapsack problem, 426, 428 ex.
- free agent, 411 pr.
- freeing of objects, 243–244
- free list, 243
- FREE-OBJECT, 244
- free tree, 1172–1176
- frequency domain, 898
- full binary tree, 1178, 1180 ex.
  - relation to optimal code, 430
- full node, 489
- full rank, 1223
- full walk of a tree, 1114
- fully parenthesized matrix-chain product, 370
- fully polynomial-time approximation scheme, 1107
  - for subset sum, 1128–1134, 1139
- function, 1166–1168
  - Ackermann's, 585
  - basis, 835
  - convex, 1199
  - final-state, 996
  - hash, *see* hash function
  - linear, 26, 845
  - objective, 664, 847, 851
  - potential, 459
  - prefix, 1003–1004
  - quadratic, 27
  - reduction, 1067
  - suffix, 996
  - transition, 995, 1001–1002, 1012 ex.
- functional iteration, 58
- fundamental theorem of linear programming, 892
- furthest-in-future strategy, 449 pr.
- fusion tree, 212, 483
- fuzzy sorting, 189 pr.
- Gabow's scaling algorithm for single-source shortest paths, 679 pr.
- gap character, 989 ex., 1002 ex.
- gap heuristic, 760 ex., 766
- garbage collection, 151, 243
- gate, 1070
- Gaussian elimination, 819, 842
- gcd, *see* greatest common divisor
- general number-field sieve, 984
- generating function, 108 pr.
- generator
  - of a subgroup, 944
  - of  $\mathbb{Z}_n^*$ , 955
- GENERIC-MST, 626
- GENERIC-PUSH-RELABEL, 741
- generic push-relabel algorithm, 740–748
- geometric distribution, 1202–1203
  - and balls and bins, 134
- geometric series, 1147
- geometry, computational, 1014–1047
- $GF(2)$ , 1227 pr.
- gift wrapping, 1037, 1047

- global variable, 21
- Goldberg's algorithm, *see* push-relabel algorithm
- golden ratio ( $\phi$ ), 59, 108 pr.
- gossiping, 478
- GRAFT, 583 pr.
- Graham's scan, 1030–1036, 1047
- GRAHAM-SCAN, 1031
- graph, 1168–1173
  - adjacency-list representation of, 590
  - adjacency-matrix representation of, 591
  - algorithms for, 587–766
  - and asymptotic notation, 588
  - attributes of, 588, 592
  - breadth-first search of, 594–602, 623
  - coloring of, 1103 pr.
  - complement of, 1090
  - component, 617
  - constraint, 666–668
  - dense, 589
  - depth-first search of, 603–612, 623
  - dynamic, 562 n.
  - $\epsilon$ -dense, 706 pr.
  - hamiltonian, 1061
  - incidence matrix of, 448 pr., 593 ex.
  - interval, 422 ex.
  - nonhamiltonian, 1061
  - shortest path in, 597
  - singly connected, 612 ex.
  - sparse, 589
  - static, 562 n.
  - subproblem, 367–368
  - tour of, 1096
  - weighted, 591
  - see also* directed acyclic graph, directed graph, flow network, undirected graph, tree
- graphic matroid, 437–438, 642
- GRAPH-ISOMORPHISM, 1065 ex.
- gray vertex, 594, 603
- greatest common divisor (gcd), 929–930, 933 ex.
  - binary gcd algorithm for, 981 pr.
  - Euclid's algorithm for, 933–939, 981 pr., 983
  - with more than two arguments, 939 ex.
  - recursion theorem for, 934
- greedoid, 450
- GREEDY, 440
  - GREEDY-ACTIVITY-SELECTOR, 421
  - greedy algorithm, 414–450
    - for activity selection, 415–422
    - for coin changing, 446 pr.
    - compared with dynamic programming, 381, 390 ex., 418, 423–427
    - Dijkstra's algorithm, 658–664
    - elements of, 423–428
    - for fractional knapsack problem, 426
    - greedy-choice property in, 424–425
    - for Huffman code, 428–437
    - Kruskal's algorithm, 631–633
    - and matroids, 437–443
    - for minimum spanning tree, 631–638
    - for multithreaded scheduling, 781–783
    - for off-line caching, 449 pr.
    - optimal substructure in, 425
    - Prim's algorithm, 634–636
    - for set cover, 1117–1122, 1139
    - for task scheduling, 443–446, 447–448 pr.
    - on a weighted matroid, 439–442
    - for weighted set cover, 1135 pr.
  - greedy-choice property, 424–425
    - of activity selection, 417–418
    - of Huffman codes, 433–434
    - of a weighted matroid, 441
  - greedy scheduler, 782
  - GREEDY-SET-COVER, 1119
  - grid, 760 pr.
  - group, 939–946
    - cyclic, 955
    - operator ( $\oplus$ ), 939
  - guessing the solution, in the substitution method, 84–85
  - half 3-CNF satisfiability, 1101 ex.
  - half-open interval, 348
  - Hall's theorem, 735 ex.
  - halting problem, 1048
  - halving lemma, 908
  - HAM-CYCLE, 1062
  - hamiltonian cycle, 1049, 1061, 1091–1096, 1105
  - hamiltonian graph, 1061
  - hamiltonian path, 1066 ex., 1101 ex.
  - HAM-PATH, 1066 ex.
  - handle, 163, 507
  - handshaking lemma, 1172 ex.

- harmonic number, 1147, 1153–1154
- harmonic series, 1147, 1153–1154
- HASH-DELETE, 277 ex.
- hash function, 256, 262–269
  - auxiliary, 272
  - collision-resistant, 964
  - division method for, 263, 268–269 ex.
  - $\epsilon$ -universal, 269 ex.
  - multiplication method for, 263–264
  - universal, 265–268
- hashing, 253–285
  - with chaining, 257–260, 283 pr.
  - double, 272–274, 277 ex.
  - $k$ -universal, 284 pr.
  - in memoization, 365, 387
  - with open addressing, 269–277
  - perfect, 277–282, 285
  - to replace adjacency lists, 593 ex.
  - universal, 265–268
- HASH-INSERT, 270, 277 ex.
- HASH-SEARCH, 271, 277 ex.
- hash table, 256–261
  - dynamic, 471 ex.
  - secondary, 278
  - see also* hashing
- hash value, 256
- hat-check problem, 122 ex.
- head
  - in a disk drive, 485
  - of a linked list, 236
  - of a queue, 234
- heap, 151–169
  - analyzed by potential method, 462 ex.
  - binomial, 527 pr.
  - building, 156–159, 166 pr.
  - compared with Fibonacci heaps, 506–507
  - $d$ -ary, 167 pr., 706 pr.
  - deletion from, 166 ex.
  - in Dijkstra’s algorithm, 662
  - extracting the maximum key from, 163
  - Fibonacci, *see* Fibonacci heap
  - as garbage-collected storage, 151
  - height of, 153
  - in Huffman’s algorithm, 433
  - to implement a mergeable heap, 506
  - increasing a key in, 163–164
  - insertion into, 164
  - in Johnson’s algorithm, 704
  - max-heap, 152
  - maximum key of, 163
  - mergeable, *see* mergeable heap
  - min-heap, 153
  - in Prim’s algorithm, 636
  - as a priority queue, 162–166
  - relaxed, 530
  - running times of operations on, 506 fig.
  - and treaps, 333 pr.
  - 2-3-4, 529 pr.
- HEAP-DECREASE-KEY, 165 ex.
- HEAP-DELETE, 166 ex.
- HEAP-EXTRACT-MAX, 163
- HEAP-EXTRACT-MIN, 165 ex.
- HEAP-INCREASE-KEY, 164
- HEAP-MAXIMUM, 163
- HEAP-MINIMUM, 165 ex.
- heap property, 152
  - maintenance of, 154–156
  - vs. binary-search-tree property, 289 ex.
- heapsort, 151–169
- HEAPSORT, 160
- heel, 602 ex.
- height
  - of a binomial tree, 527 pr.
  - black-, 309
  - of a B-tree, 489–490
  - of a  $d$ -ary heap, 167 pr.
  - of a decision tree, 193
  - exponential, 300
  - of a heap, 153
  - of a node in a heap, 153, 159 ex.
  - of a node in a tree, 1177
  - of a red-black tree, 309
  - of a tree, 1177
- height-balanced tree, 333 pr.
- height function, in push-relabel algorithms, 738
- hereditary family of subsets, 437
- Hermitian matrix, 832 ex.
- high endpoint of an interval, 348
- high function, 537, 546
- HIRE-ASSISTANT, 115
- hiring problem, 114–115, 123–124, 145
  - on-line, 139–141
  - probabilistic analysis of, 120–121
- hit
  - cache, 449 pr.
  - spurious, 991

- HOARE-PARTITION, 185 pr.  
 HOPCROFT-KARP, 764 pr.  
 Hopcroft-Karp bipartite matching algorithm,  
     763 pr.  
 horizontal ray, 1021 ex.  
 Horner's rule, 41 pr., 900  
     in the Rabin-Karp algorithm, 990  
 HUFFMAN, 431  
 Huffman code, 428–437, 450  
 hull, convex, 8, 1029–1039, 1046 pr.  
 Human Genome Project, 6  
 hyperedge, 1172  
 hypergraph, 1172  
     and bipartite graphs, 1173 ex.
- ideal parallel computer, 779  
 idempotency laws for sets, 1159  
 identity, 939  
 identity matrix, 1218  
**if**, in pseudocode, 20  
 image, 1167  
 image compression, 409 pr., 413  
 inadmissible edge, 749  
 incidence, 1169  
 incidence matrix  
     and difference constraints, 666  
     of a directed graph, 448 pr., 593 ex.  
     of an undirected graph, 448 pr.  
 inclusion and exclusion, 1163 ex.  
 incomplete step, 782  
 INCREASE-KEY, 162  
 increasing a key, in a max-heap, 163–164  
 INCREMENT, 454  
 incremental design method, 29  
     for finding the convex hull, 1030  
 in-degree, 1169  
 indentation in pseudocode, 20  
 independence  
     of events, 1192–1193, 1195 ex.  
     of random variables, 1197  
     of subproblems in dynamic programming,  
         383–384  
 independent family of subsets, 437  
 independent set, 1101 pr.  
     of tasks, 444  
 independent strands, 789  
 index function, 537, 546  
 index of an element of  $\mathbb{Z}_n^*$ , 955
- indicator random variable, 118–121  
     in analysis of expected height of a randomly  
         built binary search tree, 300–303  
     in analysis of inserting into a treap, 333 pr.  
     in analysis of streaks, 138–139  
     in analysis of the birthday paradox, 132–133  
     in approximation algorithm for  
         MAX-3-CNF satisfiability, 1124  
     in bounding the right tail of the binomial  
         distribution, 1212–1213  
     in bucket sort analysis, 202–204  
     expected value of, 118  
     in hashing analysis, 259–260  
     in hiring-problem analysis, 120–121  
     and linearity of expectation, 119  
     in quicksort analysis, 182–184, 187 pr.  
     in randomized-selection analysis, 217–219,  
         226 pr.  
     in universal-hashing analysis, 265–266  
 induced subgraph, 1171  
 inequality constraint, 852  
     and equality constraints, 853  
 inequality, linear, 846  
 infeasible linear program, 851  
 infeasible solution, 851  
 infinite sequence, 1166  
 infinite set, 1161  
 infinite sum, 1145  
 infinity, arithmetic with, 650  
 INITIALIZE-PREFLOW, 740  
 INITIALIZE-SIMPLEX, 871, 887  
 INITIALIZE-SINGLE-SOURCE, 648  
 initial strand, 779  
 injective function, 1167  
 inner product, 1222  
 inorder tree walk, 287, 293 ex., 342  
 INORDER-TREE-WALK, 288  
 in-place sorting, 17, 148, 206 pr.  
 input  
     to an algorithm, 5  
     to a combinational circuit, 1071  
     distribution of, 116, 122  
     to a logic gate, 1070  
     size of, 25  
 input alphabet, 995  
 INSERT, 162, 230, 463 ex., 505  
 insertion  
     into binary search trees, 294–295

- into a bit vector with a superimposed binary tree, 534
- into a bit vector with a superimposed tree of constant height, 534
- into B-trees, 493–497
- into chained hash tables, 258
- into  $d$ -ary heaps, 167 pr.
- into direct-address tables, 254
- into dynamic tables, 464–467
- elementary, 465
- into Fibonacci heaps, 510–511
- into heaps, 164
- into interval trees, 349
- into linked lists, 237–238
- into open-address hash tables, 270
- into order-statistic trees, 343
- into proto van Emde Boas structures, 544
- into queues, 234
- into red-black trees, 315–323
- into stacks, 232
- into sweep-line statuses, 1024
- into treaps, 333 pr.
- into 2-3-4 heaps, 529 pr.
- into van Emde Boas trees, 552–554
- into Young tableaux, 167 pr.
- insertion sort, 12, 16–20, 25–27
  - in bucket sort, 201–204
  - compared with merge sort, 14 ex.
  - compared with quicksort, 178 ex.
  - decision tree for, 192 fig.
  - in merge sort, 39 pr.
  - in quicksort, 185 ex.
  - using binary search, 39 ex.
- INSERTION-SORT, 18, 26, 208 pr.
- instance
  - of an abstract problem, 1051, 1054
  - of a problem, 5
- instructions of the RAM model, 23
- integer data type, 23
- integer linear programming, 850, 895 pr., 1101 ex.
- integers ( $\mathbb{Z}$ ), 1158
- integer-valued flow, 733
- integrality theorem, 734
- integral, to approximate summations, 1154–1156
- integration of a series, 1147
- interior of a polygon, 1020 ex.
- interior-point method, 850, 897
- intermediate vertex, 693
- internal node, 1176
- internal path length, 1180 ex.
- interpolation by a cubic spline, 840 pr.
- interpolation by a polynomial, 901, 906 ex.
  - at complex roots of unity, 912–913
- intersection
  - of chords, 345 ex.
  - determining, for a set of line segments, 1021–1029, 1047
  - determining, for two line segments, 1017–1019
  - of languages, 1058
  - of sets ( $\cap$ ), 1159
- interval, 348
  - fuzzy sorting of, 189 pr.
- INTERVAL-DELETE, 349
- interval graph, 422 ex.
- INTERVAL-INSERT, 349
- INTERVAL-SEARCH, 349, 351
- INTERVAL-SEARCH-EXACTLY, 354 ex.
- interval tree, 348–354
- interval trichotomy, 348
- intractability, 1048
- invalid shift, 985
- inventory planning, 411 pr.
- inverse
  - of a bijective function, 1167
  - in a group, 940
  - of a matrix, 827–831, 842, 1223, 1225 ex.
  - multiplicative, modulo  $n$ , 949
- inversion
  - in a self-organizing list, 476 pr.
  - in a sequence, 41 pr., 122 ex., 345 ex.
- inverter, 1070
- invertible matrix, 1223
- isolated vertex, 1169
- isomorphic graphs, 1171
- iterated function, 63 pr.
- iterated logarithm function, 58–59
- ITERATIVE-FFT, 917
- ITERATIVE-TREE-SEARCH, 291
- iter function, 577
- Jarvis's march, 1037–1038, 1047
- Jensen's inequality, 1199
- JOHNSON, 704

- Johnson's algorithm, 700–706
- joining  
   of red-black trees, 332 pr.  
   of 2-3-4 trees, 503 pr.
- joint probability density function, 1197
- Josephus permutation, 355 pr.
- Karmarkar's algorithm, 897
- Karp's minimum mean-weight cycle algorithm,  
   680 pr.
- $k$ -ary tree, 1179
- $k$ -CNF, 1049
- $k$ -coloring, 1103 pr., 1180 pr.
- $k$ -combination, 1185
- $k$ -conjunctive normal form, 1049
- kernel of a polygon, 1038 ex.
- key, 16, 147, 162, 229  
   dummy, 397  
   interpreted as a natural number, 263  
   median, of a B-tree node, 493  
   public, 959, 962  
   secret, 959, 962  
   static, 277
- keywords, in pseudocode, 20–22  
   multithreaded, 774, 776–777, 785–786
- “killer adversary” for quicksort, 190
- Kirchhoff's current law, 708
- Kleene star (\*), 1058
- KMP algorithm, 1002–1013
- KMP-MATCHER, 1005
- knapsack problem  
   fractional, 426, 428 ex.  
   0-1, 425, 427 ex., 1137 pr., 1139
- $k$ -neighbor tree, 338
- knot, of a spline, 840 pr.
- Knuth-Morris-Pratt algorithm, 1002–1013
- $k$ -permutation, 126, 1184
- Kraft inequality, 1180 ex.
- Kruskal's algorithm, 631–633, 642  
   with integer edge weights, 637 ex.
- $k$ -sorted, 207 pr.
- $k$ -string, 1184
- $k$ -subset, 1161
- $k$ -substring, 1184
- $k$ th power, 933 ex.
- $k$ -universal hashing, 284 pr.
- Lagrange's formula, 902
- Lagrange's theorem, 944
- Lamé's theorem, 936
- language, 1057  
   completeness of, 1077 ex.  
   proving NP-completeness of, 1078–1079  
   verification of, 1063
- last-in, first-out, 232  
   *see also* stack
- late task, 444
- layers  
   convex, 1044 pr.  
   maximal, 1045 pr.
- LCA, 584 pr.
- lcm (least common multiple), 939 ex.
- LCS, 7, 390–397, 413
- LCS-LENGTH, 394
- leading submatrix, 833, 839 ex.
- leaf, 1176
- least common ancestor, 584 pr.
- least common multiple, 939 ex.
- least-squares approximation, 835–839
- leaving a vertex, 1169
- leaving variable, 867
- LEFT, 152
- left child, 1178
- left-child, right-sibling representation, 246,  
   249 ex.
- LEFT-ROTATE, 313, 353 ex.
- left rotation, 312
- left spine, 333 pr.
- left subtree, 1178
- Legendre symbol ( $\frac{a}{p}$ ), 982 pr.
- length  
   of a path, 1170  
   of a sequence, 1166  
   of a spine, 333 pr.  
   of a string, 986, 1184
- level  
   of a function, 573  
   of a tree, 1177
- level function, 576
- lexicographically less than, 304 pr.
- lexicographic sorting, 304 pr.
- lg (binary logarithm), 56
- lg\* (iterated logarithm function), 58–59
- lg <sup>$k$</sup>  (exponentiation of logarithms), 56
- lg lg (composition of logarithms), 56
- LIFO (last-in, first-out), 232

- see also* stack
- light edge, 626
- linear constraint, 846
- linear dependence, 1223
- linear equality, 845
- linear equations
  - solving modular, 946–950
  - solving systems of, 813–827
  - solving tridiagonal systems of, 840 pr.
- linear function, 26, 845
- linear independence, 1223
- linear inequality, 846
- linear-inequality feasibility problem, 894 pr.
- linearity of expectation, 1198
  - and indicator random variables, 119
- linearity of summations, 1146
- linear order, 1165
- linear permutation, 1229 pr.
- linear probing, 272
- linear programming, 7, 843–897
  - algorithms for, 850
  - applications of, 849
  - duality in, 879–886
  - ellipsoid algorithm for, 850, 897
  - finding an initial solution in, 886–891
  - fundamental theorem of, 892
  - interior-point methods for, 850, 897
  - Karmarkar’s algorithm for, 897
  - and maximum flow, 860–861
  - and minimum-cost circulation, 896 pr.
  - and minimum-cost flow, 861–862
  - and minimum-cost multicommodity flow, 864 ex.
  - and multicommodity flow, 862–863
  - simplex algorithm for, 864–879, 896
  - and single-pair shortest path, 859–860
  - and single-source shortest paths, 664–670, 863 ex.
  - slack form for, 854–857
  - standard form for, 850–854
  - see also* integer linear programming, 0-1 integer programming
- linear-programming relaxation, 1125
- linear search, 22 ex.
- linear speedup, 780
- line segment, 1015
  - comparable, 1022
  - determining turn of, 1017
  - determining whether any intersect, 1021–1029, 1047
  - determining whether two intersect, 1017–1019
- link
  - of binomial trees, 527 pr.
  - of Fibonacci-heap roots, 513
  - of trees in a disjoint-set forest, 570–571
- LINK, 571
- linked list, 236–241
  - compact, 245 ex., 250 pr.
  - deletion from, 238
  - to implement disjoint sets, 564–568
  - insertion into, 237–238
  - neighbor list, 750
  - searching, 237, 268 ex.
  - self-organizing, 476 pr.
- list, *see* linked list
- LIST-DELETE, 238
- LIST-DELETE’, 238
- LIST-INSERT, 238
- LIST-INSERT’, 240
- LIST-SEARCH, 237
- LIST-SEARCH’, 239
- literal, 1082
- little-oh notation, 50–51, 64
- little-omega notation, 51
- $L_m$ -distance, 1044 ex.
- ln (natural logarithm), 56
- load factor
  - of a dynamic table, 463
  - of a hash table, 258
- load instruction, 23
- local variable, 21
- logarithm function (log), 56–57
  - discrete, 955
  - iterated ( $\lg^*$ ), 58–59
- logical parallelism, 777
- logic gate, 1070
- longest common subsequence, 7, 390–397, 413
- longest palindrome subsequence, 405 pr.
- LONGEST-PATH, 1060 ex.
- LONGEST-PATH-LENGTH, 1060 ex.
- longest simple cycle, 1101 ex.
- longest simple path, 1048
  - in an unweighted graph, 382
  - in a weighted directed acyclic graph, 404 pr.
- LOOKUP-CHAIN, 388

- loop, in pseudocode, 20
  - parallel, 785–787
- loop invariant, 18–19
  - for breadth-first search, 595
  - for building a heap, 157
  - for consolidating the root list of a Fibonacci heap, 517
  - for determining the rank of an element in an order-statistic tree, 342
  - for Dijkstra’s algorithm, 660
  - and **for** loops, 19 n.
  - for the generic minimum-spanning-tree method, 625
  - for the generic push-relabel algorithm, 743
  - for Graham’s scan, 1034
  - for heapsort, 160 ex.
  - for Horner’s rule, 41 pr.
  - for increasing a key in a heap, 166 ex.
  - initialization of, 19
  - for insertion sort, 18
  - maintenance of, 19
  - for merging, 32
  - for modular exponentiation, 957
  - origin of, 42
  - for partitioning, 171
  - for Prim’s algorithm, 636
  - for the Rabin-Karp algorithm, 993
  - for randomly permuting an array, 127, 128 ex.
  - for red-black tree insertion, 318
  - for the relabel-to-front algorithm, 755
  - for searching an interval tree, 352
  - for the simplex algorithm, 872
  - for string-matching automata, 998, 1000
  - and termination, 19
- low endpoint of an interval, 348
- lower bounds
  - on approximations, 1140
  - asymptotic, 48
  - for average sorting, 207 pr.
  - on binomial coefficients, 1186
  - for comparting water jugs, 206 pr.
  - for convex hull, 1038 ex., 1047
  - for disjoint-set data structures, 585
  - for finding the minimum, 214
  - for finding the predecessor, 560
  - for length of an optimal traveling-salesman tour, 1112–1115
  - for median finding, 227
  - for merging, 208 pr.
  - for minimum-weight vertex cover, 1124–1126
  - for multithreaded computations, 780
  - and potential functions, 478
  - for priority-queue operations, 531
  - and recurrences, 67
  - for simultaneous minimum and maximum, 215 ex.
  - for size of an optimal vertex cover, 1110, 1135 pr.
  - for sorting, 191–194, 205 pr., 211, 531
  - for streaks, 136–138, 142 ex.
  - on summations, 1152, 1154
- lower median, 213
- lower square root ( $\lfloor \sqrt{\phantom{x}} \rfloor$ ), 546
- lower-triangular matrix, 1219, 1222 ex., 1225 ex.
- low function, 537, 546
- LU decomposition, 806 pr., 819–822
- LU-DECOMPOSITION, 821
- LUP decomposition, 806 pr., 815
  - computation of, 822–825
  - of a diagonal matrix, 827 ex.
  - in matrix inversion, 828
  - and matrix multiplication, 832 ex.
  - of a permutation matrix, 827 ex.
  - use of, 815–819
- LUP-DECOMPOSITION, 824
- LUP-SOLVE, 817
- main memory, 484
- MAKE-HEAP, 505
- MAKE-SET, 561
  - disjoint-set-forest implementation of, 571
  - linked-list implementation of, 564
- makespan, 1136 pr.
- MAKE-TREE, 583 pr.
- Manhattan distance, 225 pr., 1044 ex.
- marked node, 508, 519–520
- Markov’s inequality, 1201 ex.
- master method for solving a recurrence, 93–97
- master theorem, 94
  - proof of, 97–106
- matched vertex, 732
- matching
  - bipartite, 732, 763 pr.

- maximal, 1110, 1135 pr.
- maximum, 1135 pr.
- and maximum flow, 732–736, 747 ex.
- perfect, 735 ex.
- of strings, 985–1013
- weighted bipartite, 530
- matric matroid, 437
- matrix, 1217–1229
  - addition of, 1220
  - adjacency, 591
  - conjugate transpose of, 832 ex.
  - determinant of, 1224–1225
  - diagonal, 1218
  - Hermitian, 832 ex.
  - identity, 1218
  - incidence, 448 pr., 593 ex.
  - inversion of, 806 pr., 827–831, 842
  - lower-triangular, 1219, 1222 ex., 1225 ex.
  - multiplication of, *see* matrix multiplication
  - negative of, 1220
  - permutation, 1220, 1222 ex.
  - predecessor, 685
  - product of, with a vector, 785–787, 789–790, 792 ex.
  - pseudoinverse of, 837
  - scalar multiple of, 1220
  - subtraction of, 1221
  - symmetric, 1220
  - symmetric positive-definite, 832–835, 842
  - Toeplitz, 921 pr.
  - transpose of, 797 ex., 1217
  - transpose of, multithreaded, 792 ex.
  - tridiagonal, 1219
  - unit lower-triangular, 1219
  - unit upper-triangular, 1219
  - upper-triangular, 1219, 1225 ex.
  - Vandermonde, 902, 1226 pr.
- matrix-chain multiplication, 370–378
- MATRIX-CHAIN-MULTIPLY
- MATRIX-CHAIN-ORDER, 375
- matrix multiplication, 75–83, 1221
  - for all-pairs shortest paths, 686–693, 706–707
  - boolean, 832 ex.
  - and computing the determinant, 832 ex.
  - divide-and-conquer method for, 76–83
  - and LUP decomposition, 832 ex.
  - and matrix inversion, 828–831, 842
  - multithreaded algorithm for, 792–797, 806 pr.
  - Pan’s method for, 82 ex.
  - Strassen’s algorithm for, 79–83, 111–112
- MATRIX-MULTIPLY, 371
- matrix-vector multiplication, multithreaded, 785–787, 792 ex.
  - with race, 789–790
- matroid, 437–443, 448 pr., 450, 642
  - for task scheduling, 443–446
- MAT-VEC, 785
- MAT-VEC-MAIN-LOOP, 786
- MAT-VEC-WRONG, 790
- MAX-CNF satisfiability, 1127 ex.
- MAX-CUT problem, 1127 ex.
- MAX-FLOW-BY-SCALING, 763 pr.
- max-flow min-cut theorem, 723
- max-heap, 152
  - building, 156–159
  - $d$ -ary, 167 pr.
  - deletion from, 166 ex.
  - extracting the maximum key from, 163
  - in heapsort, 159–162
  - increasing a key in, 163–164
  - insertion into, 164
  - maximum key of, 163
  - as a max-priority queue, 162–166
  - mergeable, 250 n., 481 n., 505 n.
- MAX-HEAPIFY, 154
- MAX-HEAP-INSERT, 164
  - building a heap with, 166 pr.
- max-heap property, 152
  - maintenance of, 154–156
- maximal element, of a partially ordered set, 1165
- maximal layers, 1045 pr.
- maximal matching, 1110, 1135 pr.
- maximal point, 1045 pr.
- maximal subset, in a matroid, 438
- maximization linear program, 846
  - and minimization linear programs, 852
- maximum, 213
  - in binary search trees, 291
  - of a binomial distribution, 1207 ex.
  - in a bit vector with a superimposed binary tree, 533
  - in a bit vector with a superimposed tree of constant height, 535

- finding, 214–215
  - in heaps, 163
  - in order-statistic trees, 347 ex.
  - in proto van Emde Boas structures, 544 ex.
  - in red-black trees, 311
  - in van Emde Boas trees, 550
- MAXIMUM, 162–163, 230
- maximum bipartite matching, 732–736, 747 ex., 766
  - Hopcroft-Karp algorithm for, 763 pr.
- maximum degree, in a Fibonacci heap, 509, 523–526
- maximum flow, 708–766
  - Edmonds-Karp algorithm for, 727–730
  - Ford-Fulkerson method for, 714–731, 765
  - as a linear program, 860–861
  - and maximum bipartite matching, 732–736, 747 ex.
  - push-relabel algorithms for, 736–760, 765
  - relabel-to-front algorithm for, 748–760
  - scaling algorithm for, 762 pr., 765
  - updating, 762 pr.
- maximum matching, 1135 pr.
- maximum spanning tree, 1137 pr.
- maximum-subarray problem, 68–75, 111
- max-priority queue, 162
- MAX-3-CNF satisfiability, 1123–1124, 1139
- MAYBE-MST-A, 641 pr.
- MAYBE-MST-B, 641 pr.
- MAYBE-MST-C, 641 pr.
- mean, *see* expected value
- mean weight of a cycle, 680 pr.
- median, 213–227
  - multithreaded algorithm for, 805 ex.
  - of sorted lists, 223 ex.
  - of two sorted lists, 804 ex.
  - weighted, 225 pr.
- median key, of a B-tree node, 493
- median-of-3 method, 188 pr.
- member of a set ( $\in$ ), 1158
- membership
  - in proto van Emde Boas structures, 540–541
  - in Van Emde Boas trees, 550
- memoization, 365, 387–389
- MEMOIZED-CUT-ROD, 365
- MEMOIZED-CUT-ROD-AUX, 366
- MEMOIZED-MATRIX-CHAIN, 388
- memory, 484
  - memory hierarchy, 24
- MERGE, 31
- mergeable heap, 481, 505
  - binomial heaps, 527 pr.
  - linked-list implementation of, 250 pr.
  - relaxed heaps, 530
  - running times of operations on, 506 fig.
  - 2-3-4 heaps, 529 pr.
  - see also* Fibonacci heap
- mergeable max-heap, 250 n., 481 n., 505 n.
- mergeable min-heap, 250 n., 481 n., 505
- MERGE-LISTS, 1129
- merge sort, 12, 30–37
  - compared with insertion sort, 14 ex.
  - multithreaded algorithm for, 797–805, 812
  - use of insertion sort in, 39 pr.
- MERGE-SORT, 34
- MERGE-SORT', 797
- merging
  - of  $k$  sorted lists, 166 ex.
  - lower bounds for, 208 pr.
  - multithreaded algorithm for, 798–801
  - of two sorted arrays, 30
- MILLER-RABIN, 970
- Miller-Rabin primality test, 968–975, 983
- MIN-GAP, 354 ex.
- min-heap, 153
  - analyzed by potential method, 462 ex.
  - building, 156–159
  - $d$ -ary, 706 pr.
  - in Dijkstra's algorithm, 662
  - in Huffman's algorithm, 433
  - in Johnson's algorithm, 704
  - mergeable, 250 n., 481 n., 505
  - as a min-priority queue, 165 ex.
  - in Prim's algorithm, 636
- MIN-HEAPIFY, 156 ex.
- MIN-HEAP-INSERT, 165 ex.
- min-heap ordering, 507
- min-heap property, 153, 507
  - maintenance of, 156 ex.
  - in treaps, 333 pr.
  - vs. binary-search-tree property, 289 ex.
- minimization linear program, 846
  - and maximization linear programs, 852
- minimum, 213
  - in binary search trees, 291

- in a bit vector with a superimposed binary tree, 533
- in a bit vector with a superimposed tree of constant height, 535
- in B-trees, 497 ex.
- in Fibonacci heaps, 511
- finding, 214–215
- off-line, 582 pr.
- in order-statistic trees, 347 ex.
- in proto van Emde Boas structures, 541–542
- in red-black trees, 311
- in 2-3-4 heaps, 529 pr.
- in van Emde Boas trees, 550
- MINIMUM, 162, 214, 230, 505
- minimum-cost circulation, 896 pr.
- minimum-cost flow, 861–862
- minimum-cost multicommodity flow, 864 ex.
- minimum-cost spanning tree, *see* minimum spanning tree
- minimum cut, 721, 731 ex.
- minimum degree, of a B-tree, 489
- minimum mean-weight cycle, 680 pr.
- minimum node, of a Fibonacci heap, 508
- minimum path cover, 761 pr.
- minimum spanning tree, 624–642
  - in approximation algorithm for traveling-salesman problem, 1112
  - Borůvka’s algorithm for, 641
  - on dynamic graphs, 637 ex.
  - generic method for, 625–630
  - Kruskal’s algorithm for, 631–633
  - Prim’s algorithm for, 634–636
  - relation to matroids, 437, 439–440
  - second-best, 638 pr.
- minimum-weight spanning tree, *see* minimum spanning tree
- minimum-weight vertex cover, 1124–1127, 1139
- minor of a matrix, 1224
- min-priority queue, 162
  - in constructing Huffman codes, 431
  - in Dijkstra’s algorithm, 661
  - in Prim’s algorithm, 634, 636
- miss, 449 pr.
- missing child, 1178
- mod, 54, 928
- modifying operation, 230
- modular arithmetic, 54, 923 pr., 939–946
- modular equivalence, 54, 1165 ex.
- modular exponentiation, 956
- MODULAR-EXPONENTIATION, 957
- modular linear equations, 946–950
- MODULAR-LINEAR-EQUATION-SOLVER, 949
- modulo, 54, 928
- Monge array, 110 pr.
- monotone sequence, 168
- monotonically decreasing, 53
- monotonically increasing, 53
- Monty Hall problem, 1195 ex.
- move-to-front heuristic, 476 pr., 478
- MST-KRUSKAL, 631
- MST-PRIM, 634
- MST-REDUCE, 639 pr.
- much-greater-than ( $\gg$ ), 574
- much-less-than ( $\ll$ ), 783
- multicommodity flow, 862–863
  - minimum-cost, 864 ex.
- multicore computer, 772
- multidimensional fast Fourier transform, 921 pr.
- multigraph, 1172
  - converting to equivalent undirected graph, 593 ex.
- multiple, 927
  - of an element modulo  $n$ , 946–950
  - least common, 939 ex.
  - scalar, 1220
- multiple assignment, 21
- multiple sources and sinks, 712
- multiplication
  - of complex numbers, 83 ex.
  - divide-and-conquer method for, 920 pr.
  - of matrices, *see* matrix multiplication
  - of a matrix chain, 370–378
  - matrix-vector, multithreaded, 785–787, 789–790, 792 ex.
  - modulo  $n$  ( $\cdot_n$ ), 940
  - of polynomials, 899
- multiplication method, 263–264
- multiplicative group modulo  $n$ , 941
- multiplicative inverse, modulo  $n$ , 949
- multiply instruction, 23
- MULTIPOP, 453
- multiprocessor, 772
- MULTIPUSH, 456 ex.

- multiset, 1158 n.
- multithreaded algorithm, 10, 772–812
  - for computing Fibonacci numbers, 774–780
  - for fast Fourier transform, 804 ex.
  - Floyd-Warshall algorithm, 797 ex.
  - for LU decomposition, 806 pr.
  - for LUP decomposition, 806 pr.
  - for matrix inversion, 806 pr.
  - for matrix multiplication, 792–797, 806 pr.
  - for matrix transpose, 792 ex., 797 ex.
  - for matrix-vector product, 785–787, 789–790, 792 ex.
  - for median, 805 ex.
  - for merge sorting, 797–805, 812
  - for merging, 798–801
  - for order statistics, 805 ex.
  - for partitioning, 804 ex.
  - for prefix computation, 807 pr.
  - for quicksort, 811 pr.
  - for reduction, 807 pr.
  - for a simple stencil calculation, 809 pr.
  - for solving systems of linear equations, 806 pr.
  - Strassen’s algorithm, 795–796
- multithreaded composition, 784 fig.
- multithreaded computation, 777
- multithreaded scheduling, 781–783
- mutually exclusive events, 1190
- mutually independent events, 1193
- $\mathbb{N}$  (set of natural numbers), 1158
- naive algorithm, for string matching, 988–990
- NAIVE-STRING-MATCHER, 988
- natural cubic spline, 840 pr.
- natural numbers ( $\mathbb{N}$ ), 1158
  - keys interpreted as, 263
- negative of a matrix, 1220
- negative-weight cycle
  - and difference constraints, 667
  - and relaxation, 677 ex.
  - and shortest paths, 645, 653–654, 692 ex., 700 ex.
- negative-weight edges, 645–646
- neighbor, 1172
- neighborhood, 735 ex.
- neighbor list, 750
- nested parallelism, 776, 805 pr.
- nesting boxes, 678 pr.
- net flow across a cut, 720
- network
  - admissible, 749–750
  - flow, *see* flow network
  - residual, 715–719
  - for sorting, 811
- NEXT-TO-TOP, 1031
- NIL, 21
- node, 1176
  - see also* vertex
- nonbasic variable, 855
- nondeterministic multithreaded algorithm, 787
- nondeterministic polynomial time, 1064 n.
  - see also* NP
- nonhamiltonian graph, 1061
- noninstance, 1056 n.
- noninvertible matrix, 1223
- nonnegativity constraint, 851, 853
- nonoverlappable string pattern, 1002 ex.
- nonsaturating push, 739, 745
- nonsingular matrix, 1223
- nontrivial power, 933 ex.
- nontrivial square root of 1, modulo  $n$ , 956
- no-path property, 650, 672
- normal equation, 837
- norm of a vector, 1222
- NOT function ( $\neg$ ), 1071
- not a set member ( $\notin$ ), 1158
- not equivalent ( $\neq$ ), 54
- NOT gate, 1070
- NP (complexity class), 1049, 1064, 1066 ex., 1105
- NPC (complexity class), 1050, 1069
- NP-complete, 1050, 1069
- NP-completeness, 9–10, 1048–1105
  - of the circuit-satisfiability problem, 1070–1077
  - of the clique problem, 1086–1089, 1105
  - of determining whether a boolean formula is a tautology, 1086 ex.
  - of the formula-satisfiability problem, 1079–1081, 1105
  - of the graph-coloring problem, 1103 pr.
  - of the half 3-CNF satisfiability problem, 1101 ex.
  - of the hamiltonian-cycle problem, 1091–1096, 1105
  - of the hamiltonian-path problem, 1101 ex.

- of the independent-set problem, 1101 pr.
- of integer linear programming, 1101 ex.
- of the longest-simple-cycle problem, 1101 ex.
- proving, of a language, 1078–1079
- of scheduling with profits and deadlines, 1104 pr.
- of the set-covering problem, 1122 ex.
- of the set-partition problem, 1101 ex.
- of the subgraph-isomorphism problem, 1100 ex.
- of the subset-sum problem, 1097–1100
- of the 3-CNF-satisfiability problem, 1082–1085, 1105
- of the traveling-salesman problem, 1096–1097
- of the vertex-cover problem, 1089–1091, 1105
- of 0-1 integer programming, 1100 ex.
- NP-hard, 1069
- $n$ -set, 1161
- $n$ -tuple, 1162
- null event, 1190
- null tree, 1178
- null vector, 1224
- number-field sieve, 984
- numerical stability, 813, 815, 842
- $n$ -vector, 1218
  
- $o$ -notation, 50–51, 64
- $O$ -notation, 45 fig., 47–48, 64
- $O'$ -notation, 62 pr.
- $\tilde{O}$ -notation, 62 pr.
- object, 21
  - allocation and freeing of, 243–244
  - array implementation of, 241–246
  - passing as parameter, 21
- objective function, 664, 847, 851
- objective value, 847, 851
- oblivious compare-exchange algorithm, 208 pr.
- occurrence of a pattern, 985
- OFF-LINE-MINIMUM, 583 pr.
- off-line problem
  - caching, 449 pr.
  - least common ancestors, 584 pr.
  - minimum, 582 pr.
- Omega-notation, 45 fig., 48–49, 64
- 1-approximation algorithm, 1107
- one-pass method, 585
- one-to-one correspondence, 1167
- one-to-one function, 1167
- on-line convex-hull problem, 1039 ex.
- on-line hiring problem, 139–141
- ON-LINE-MAXIMUM, 140
- on-line multithreaded scheduler, 781
- ON-SEGMENT, 1018
- onto function, 1167
- open-address hash table, 269–277
  - with double hashing, 272–274, 277 ex.
  - with linear probing, 272
  - with quadratic probing, 272, 283 pr.
- open interval, 348
- OpenMP, 774
- optimal binary search tree, 397–404, 413
- OPTIMAL-BST, 402
- optimal objective value, 851
- optimal solution, 851
- optimal subset, of a matroid, 439
- optimal substructure
  - of activity selection, 416
  - of binary search trees, 399–400
  - in dynamic programming, 379–384
  - of the fractional knapsack problem, 426
  - in greedy algorithms, 425
  - of Huffman codes, 435
  - of longest common subsequences, 392–393
  - of matrix-chain multiplication, 373
  - of rod-cutting, 362
  - of shortest paths, 644–645, 687, 693–694
  - of unweighted shortest paths, 382
  - of weighted matroids, 442
  - of the 0-1 knapsack problem, 426
- optimal vertex cover, 1108
- optimization problem, 359, 1050, 1054
  - approximation algorithms for, 10, 1106–1140
  - and decision problems, 1051
- OR function ( $\vee$ ), 697, 1071
- order
  - of a group, 945
  - linear, 1165
  - partial, 1165
  - total, 1165
- ordered pair, 1161
- ordered tree, 1177
- order of growth, 28

- order statistics, 213–227
  - dynamic, 339–345
  - multithreaded algorithm for, 805 ex.
- order-statistic tree, 339–345
  - querying, 347 ex.
- OR gate, 1070
- origin, 1015
- or, in pseudocode, 22
- orthonormal, 842
- OS-KEY-RANK, 344 ex.
- OS-RANK, 342
- OS-SELECT, 341
- out-degree, 1169
- outer product, 1222
- output
  - of an algorithm, 5
  - of a combinational circuit, 1071
  - of a logic gate, 1070
- overdetermined system of linear equations, 814
- overflow
  - of a queue, 235
  - of a stack, 233
- overflowing vertex, 736
  - discharge of, 751
- overlapping intervals, 348
  - finding all, 354 ex.
  - point of maximum overlap, 354 pr.
- overlapping rectangles, 354 ex.
- overlapping subproblems, 384–386
- overlapping-suffix lemma, 987
  
- P (complexity class), 1049, 1055, 1059, 1061 ex., 1105
- package wrapping, 1037, 1047
- page on a disk, 486, 499 ex., 502 pr.
- pair, ordered, 1161
- pairwise disjoint sets, 1161
- pairwise independence, 1193
- pairwise relatively prime, 931
- palindrome, 405 pr.
- Pan's method for matrix multiplication, 82 ex.
- parallel algorithm, 10, 772
  - see also* multithreaded algorithm
- parallel computer, 772
  - ideal, 779
- parallel for**, in pseudocode, 785–786
- parallelism
  - logical, 777
  - of a multithreaded computation, 780
  - nested, 776
  - of a randomized multithreaded algorithm, 811 pr.
- parallel loop, 785–787, 805 pr.
- parallel-machine-scheduling problem, 1136 pr.
- parallel prefix, 807 pr.
- parallel random-access machine, 811
- parallel slackness, 781
  - rule of thumb, 783
- parallel, strands being logically in, 778
- parameter, 21
  - costs of passing, 107 pr.
- parent
  - in a breadth-first tree, 594
  - in a multithreaded computation, 776
  - in a rooted tree, 1176
- PARENT, 152
- parenthesis structure of depth-first search, 606
- parenthesis theorem, 606
- parenthesization of a matrix-chain product, 370
- parse tree, 1082
- partially ordered set, 1165
- partial order, 1165
- PARTITION, 171
- PARTITION', 186 pr.
- partition function, 361 n.
- partitioning, 171–173
  - around median of 3 elements, 185 ex.
  - Hoare's method for, 185 pr.
  - multithreaded algorithm for, 804 ex.
  - randomized, 179
- partition of a set, 1161, 1164
- Pascal's triangle, 1188 ex.
- path, 1170
  - augmenting, 719–720, 763 pr.
  - critical, 657
  - find, 569
  - hamiltonian, 1066 ex.
  - longest, 382, 1048
  - shortest, *see* shortest paths
  - simple, 1170
  - weight of, 643
- PATH, 1051, 1058
- path compression, 569
- path cover, 761 pr.
- path length, of a tree, 304 pr., 1180 ex.
- path-relaxation property, 650, 673

- pattern, in string matching, 985
  - nonoverlappable, 1002 ex.
- pattern matching, *see* string matching
- penalty, 444
- perfect hashing, 277–282, 285
- perfect linear speedup, 780
- perfect matching, 735 ex.
- permutation, 1167
  - bit-reversal, 472 pr., 918
  - Josephus, 355 pr.
  - $k$ -permutation, 126, 1184
  - linear, 1229 pr.
  - in place, 126
  - random, 124–128
  - of a set, 1184
  - uniform random, 116, 125
- permutation matrix, 1220, 1222 ex., 1226 ex.
  - LUP decomposition of, 827 ex.
- PERMUTE-BY-CYCLIC, 129 ex.
- PERMUTE-BY-SORTING, 125
- PERMUTE-WITH-ALL, 129 ex.
- PERMUTE-WITHOUT-IDENTITY, 128 ex.
- persistent data structure, 331 pr., 482
- PERSISTENT-TREE-INSERT, 331 pr.
- PERT chart, 657, 657 ex.
- P-FIB, 776
- phase, of the relabel-to-front algorithm, 758
- phi function ( $\phi(n)$ ), 943
- PISANO-DELETE, 526 pr.
- pivot
  - in linear programming, 867, 869–870, 878 ex.
  - in LU decomposition, 821
  - in quicksort, 171
- PIVOT, 869
- platter, 485
- P-MATRIX-MULTIPLY-RECURSIVE, 794
- P-MERGE, 800
- P-MERGE-SORT, 803
- pointer, 21
  - array implementation of, 241–246
  - trailing, 295
- point-value representation, 901
- polar angle, 1020 ex.
- Pollard's rho heuristic, 976–980, 980 ex., 984
- POLLARD-RHO, 976
- polygon, 1020 ex.
  - kernel of, 1038 ex.
  - star-shaped, 1038 ex.
- polylogarithmically bounded, 57
- polynomial, 55, 898
  - addition of, 898
  - asymptotic behavior of, 61 pr.
  - coefficient representation of, 900
  - derivatives of, 922 pr.
  - evaluation of, 41 pr., 900, 905 ex., 923 pr.
  - interpolation by, 901, 906 ex.
  - multiplication of, 899, 903–905, 920 pr.
  - point-value representation of, 901
- polynomial-growth condition, 113
- polynomially bounded, 55
- polynomially related, 1056
- polynomial-time acceptance, 1058
- polynomial-time algorithm, 927, 1048
- polynomial-time approximation scheme, 1107
  - for maximum clique, 1134 pr.
- polynomial-time computability, 1056
- polynomial-time decision, 1059
- polynomial-time reducibility ( $\leq_p$ ), 1067, 1077 ex.
- polynomial-time solvability, 1055
- polynomial-time verification, 1061–1066
- POP, 233, 452
- pop from a run-time stack, 188 pr.
- positional tree, 1178
- positive-definite matrix, 1225
- post-office location problem, 225 pr.
- postorder tree walk, 287
- potential function, 459
  - for lower bounds, 478
- potential method, 459–463
  - for binary counters, 461–462
  - for disjoint-set data structures, 575–581, 582 ex.
  - for dynamic tables, 466–471
  - for Fibonacci heaps, 509–512, 517–518, 520–522
  - for the generic push-relabel algorithm, 746
  - for min-heaps, 462 ex.
  - for restructuring red-black trees, 474 pr.
  - for self-organizing lists with move-to-front, 476 pr.
  - for stack operations, 460–461
- potential, of a data structure, 459
- power
  - of an element, modulo  $n$ , 954–958

- $k$ th, 933 ex.
  - nontrivial, 933 ex.
- power series, 108 pr.
- power set, 1161
- $\text{Pr} \{ \}$  (probability distribution), 1190
- PRAM, 811
- predecessor
  - in binary search trees, 291–292
  - in a bit vector with a superimposed binary tree, 534
  - in a bit vector with a superimposed tree of constant height, 535
  - in breadth-first trees, 594
  - in B-trees, 497 ex.
  - in linked lists, 236
  - in order-statistic trees, 347 ex.
  - in proto van Emde Boas structures, 544 ex.
  - in red-black trees, 311
  - in shortest-paths trees, 647
  - in Van Emde Boas trees, 551–552
- PREDECESSOR, 230
- predecessor matrix, 685
- predecessor subgraph
  - in all-pairs shortest paths, 685
  - in breadth-first search, 600
  - in depth-first search, 603
  - in single-source shortest paths, 647
- predecessor-subgraph property, 650, 676
- preemption, 447 pr.
- prefix
  - of a sequence, 392
  - of a string ( $\square$ ), 986
- prefix code, 429
- prefix computation, 807 pr.
- prefix function, 1003–1004
- prefix-function iteration lemma, 1007
- preflow, 736, 765
- preimage of a matrix, 1228 pr.
- preorder, total, 1165
- preorder tree walk, 287
- presorting, 1043
- Prim’s algorithm, 634–636, 642
  - with an adjacency matrix, 637 ex.
  - in approximation algorithm for traveling-salesman problem, 1112
  - implemented with a Fibonacci heap, 636
  - implemented with a min-heap, 636
  - with integer edge weights, 637 ex.
  - similarity to Dijkstra’s algorithm, 634, 662
  - for sparse graphs, 638 pr.
- primality testing, 965–975, 983
  - Miller-Rabin test, 968–975, 983
  - pseudoprimalty testing, 966–968
- primal linear program, 880
- primary clustering, 272
- primary memory, 484
- prime distribution function, 965
- prime number, 928
  - density of, 965–966
- prime number theorem, 965
- primitive root of  $\mathbb{Z}_n^*$ , 955
- principal root of unity, 907
- principle of inclusion and exclusion, 1163 ex.
- PRINT-ALL-PAIRS-SHORTEST-PATH, 685
- PRINT-CUT-ROD-SOLUTION, 369
- PRINT-INTERSECTING-SEGMENTS, 1028 ex.
- PRINT-LCS, 395
- PRINT-OPTIMAL-PARENS, 377
- PRINT-PATH, 601
- PRINT-SET, 572 ex.
- priority queue, 162–166
  - in constructing Huffman codes, 431
  - in Dijkstra’s algorithm, 661
  - heap implementation of, 162–166
  - lower bounds for, 531
  - max-priority queue, 162
  - min-priority queue, 162, 165 ex.
  - with monotone extractions, 168
  - in Prim’s algorithm, 634, 636
  - proto van Emde Boas structure
    - implementation of, 538–545
    - van Emde Boas tree implementation of, 531–560
  - see also* binary search tree, binomial heap, Fibonacci heap
- probabilistically checkable proof, 1105, 1140
- probabilistic analysis, 115–116, 130–142
  - of approximation algorithm for MAX-3-CNF satisfiability, 1124
  - and average inputs, 28
  - of average node depth in a randomly built binary search tree, 304 pr.
  - of balls and bins, 133–134
  - of birthday paradox, 130–133
  - of bucket sort, 201–204, 204 ex.
  - of collisions, 261 ex., 282 ex.

- of convex hull over a sparse-hulled distribution, 1046 pr.
- of file comparison, 995 ex.
- of fuzzy sorting of intervals, 189 pr.
- of hashing with chaining, 258–260
- of height of a randomly built binary search tree, 299–303
- of hiring problem, 120–121, 139–141
- of insertion into a binary search tree with equal keys, 303 pr.
- of longest-probe bound for hashing, 282 pr.
- of lower bound for sorting, 205 pr.
- of Miller-Rabin primality test, 971–975
- and multithreaded algorithms, 811 pr.
- of on-line hiring problem, 139–141
- of open-address hashing, 274–276, 277 ex.
- of partitioning, 179 ex., 185 ex., 187–188 pr.
- of perfect hashing, 279–282
- of Pollard’s rho heuristic, 977–980
- of probabilistic counting, 143 pr.
- of quicksort, 181–184, 187–188 pr., 303 ex.
- of Rabin-Karp algorithm, 994
- and randomized algorithms, 123–124
- of randomized selection, 217–219, 226 pr.
- of searching a compact list, 250 pr.
- of slot-size bound for chaining, 283 pr.
- of sorting points by distance from origin, 204 ex.
- of streaks, 135–139
- of universal hashing, 265–268
- probabilistic counting, 143 pr.
- probability, 1189–1196
- probability density function, 1196
- probability distribution, 1190
- probability distribution function, 204 ex.
- probe sequence, 270
- probing, 270, 282 pr.
  - see also* linear probing, quadratic probing, double hashing
- problem
  - abstract, 1054
  - computational, 5–6
  - concrete, 1055
  - decision, 1051, 1054
  - intractable, 1048
  - optimization, 359, 1050, 1054
  - solution to, 6, 1054–1055
  - tractable, 1048
- procedure, 6, 16–17
- product ( $\prod$ ), 1148
  - Cartesian, 1162
  - cross, 1016
  - inner, 1222
  - of matrices, 1221, 1226 ex.
  - outer, 1222
  - of polynomials, 899
  - rule of, 1184
  - scalar flow, 714 ex.
- professional wrestler, 602 ex.
- program counter, 1073
- programming, *see* dynamic programming, linear programming
- proper ancestor, 1176
- proper descendant, 1176
- proper subgroup, 944
- proper subset ( $\subset$ ), 1159
- proto van Emde Boas structure, 538–545
  - cluster in, 538
  - compared with van Emde Boas trees, 547
  - deletion from, 544
  - insertion into, 544
  - maximum in, 544 ex.
  - membership in, 540–541
  - minimum in, 541–542
  - predecessor in, 544 ex.
  - successor in, 543–544
  - summary in, 540
- PROTO-VEB-INSERT, 544
- PROTO-VEB-MEMBER, 541
- PROTO-VEB-MINIMUM, 542
- proto-vEB structure, *see* proto van Emde Boas structure
- PROTO-VEB-SUCCESSOR, 543
- prune-and-search method, 1030
- pruning a Fibonacci heap, 529 pr.
- P-SCAN-1, 808 pr.
- P-SCAN-2, 808 pr.
- P-SCAN-3, 809 pr.
- P-SCAN-DOWN, 809 pr.
- P-SCAN-UP, 809 pr.
- pseudocode, 16, 20–22
- pseudoinverse, 837
- pseudoprime, 966–968
- PSEUDOPRIME, 967
- pseudorandom-number generator, 117
- P-SQUARE-MATRIX-MULTIPLY, 793

- P-TRANSDUCE, 792 ex.
- public key, 959, 962
- public-key cryptosystem, 958–965, 983
- PUSH
  - push-relabel operation, 739
  - stack operation, 233, 452
  - push onto a run-time stack, 188 pr.
  - push operation (in push-relabel algorithms), 738–739
    - nonsaturating, 739, 745
    - saturating, 739, 745
  - push-relabel algorithm, 736–760, 765
    - basic operations in, 738–740
    - by discharging an overflowing vertex of maximum height, 760 ex.
    - to find a maximum bipartite matching, 747 ex.
    - gap heuristic for, 760 ex., 766
    - generic algorithm, 740–748
    - with a queue of overflowing vertices, 759 ex.
    - relabel-to-front algorithm, 748–760
- quadratic function, 27
- quadratic probing, 272, 283 pr.
- quadratic residue, 982 pr.
- quantile, 223 ex.
- query, 230
- queue, 232, 234–235
  - in breadth-first search, 595
  - implemented by stacks, 236 ex.
  - linked-list implementation of, 240 ex.
  - priority, *see* priority queue
  - in push-relabel algorithms, 759 ex.
- quicksort, 170–190
  - analysis of, 174–185
  - average-case analysis of, 181–184
  - compared with insertion sort, 178 ex.
  - compared with radix sort, 199
  - with equal element values, 186 pr.
  - good worst-case implementation of, 223 ex.
  - “killer adversary” for, 190
  - with median-of-3 method, 188 pr.
  - multithreaded algorithm for, 811 pr.
  - randomized version of, 179–180, 187 pr.
  - stack depth of, 188 pr.
  - tail-recursive version of, 188 pr.
  - use of insertion sort in, 185 ex.
  - worst-case analysis of, 180–181
- QUICKSORT, 171
- QUICKSORT', 186 pr.
- quotient, 928
- $\mathbb{R}$  (set of real numbers), 1158
- Rabin-Karp algorithm, 990–995, 1013
- RABIN-KARP-MATCHER, 993
- race, 787–790
- RACE-EXAMPLE, 788
- radix sort, 197–200
  - compared with quicksort, 199
- RADIX-SORT, 198
- radix tree, 304 pr.
- RAM, 23–24
- RANDOM, 117
- random-access machine, 23–24
  - parallel, 811
- randomized algorithm, 116–117, 122–130
  - and average inputs, 28
  - comparison sort, 205 pr.
  - for fuzzy sorting of intervals, 189 pr.
  - for hiring problem, 123–124
  - for insertion into a binary search tree with equal keys, 303 pr.
  - for MAX-3-CNF satisfiability, 1123–1124, 1139
  - Miller-Rabin primality test, 968–975, 983
  - multithreaded, 811 pr.
  - for partitioning, 179, 185 ex., 187–188 pr.
  - for permuting an array, 124–128
  - Pollard's rho heuristic, 976–980, 980 ex., 984
  - and probabilistic analysis, 123–124
  - quicksort, 179–180, 185 ex., 187–188 pr.
  - randomized rounding, 1139
  - for searching a compact list, 250 pr.
  - for selection, 215–220
  - universal hashing, 265–268
  - worst-case performance of, 180 ex.
- RANDOMIZED-HIRE-ASSISTANT, 124
- RANDOMIZED-PARTITION, 179
- RANDOMIZED-QUICKSORT, 179, 303 ex.
  - relation to randomly built binary search trees, 304 pr.
- randomized rounding, 1139
- RANDOMIZED-SELECT, 216
- RANDOMIZE-IN-PLACE, 126

- randomly built binary search tree, 299–303, 304 pr.
- random-number generator, 117
- random permutation, 124–128
  - uniform, 116, 125
- RANDOM-SAMPLE, 130 ex.
- random sampling, 129 ex., 179
- RANDOM-SEARCH, 143 pr.
- random variable, 1196–1201
  - indicator, *see* indicator random variable
- range, 1167
  - of a matrix, 1228 pr.
- rank
  - column, 1223
  - full, 1223
  - of a matrix, 1223, 1226 ex.
  - of a node in a disjoint-set forest, 569, 575, 581 ex.
  - of a number in an ordered set, 300, 339
  - in order-statistic trees, 341–343, 344–345 ex.
  - row, 1223
- rate of growth, 28
- ray, 1021 ex.
- RB-DELETE, 324
- RB-DELETE-FIXUP, 326
- RB-ENUMERATE, 348 ex.
- RB-INSERT, 315
- RB-INSERT-FIXUP, 316
- RB-JOIN, 332 pr.
- RB-TRANSPLANT, 323
- reachability in a graph ( $\rightsquigarrow$ ), 1170
- real numbers ( $\mathbb{R}$ ), 1158
- reconstructing an optimal solution, in dynamic programming, 387
- record, 147
- rectangle, 354 ex.
- recurrence, 34, 65–67, 83–113
  - solution by Akra-Bazzi method, 112–113
  - solution by master method, 93–97
  - solution by recursion-tree method, 88–93
  - solution by substitution method, 83–88
- recurrence equation, *see* recurrence
- recursion, 30
- recursion tree, 37, 88–93
  - in proof of master theorem, 98–100
  - and the substitution method, 91–92
- RECURSIVE-ACTIVITY-SELECTOR, 419
- recursive case, 65
- RECURSIVE-FFT, 911
- RECURSIVE-MATRIX-CHAIN, 385
- red-black tree, 308–338
  - augmentation of, 346–347
  - compared with B-trees, 484, 490
  - deletion from, 323–330
  - in determining whether any line segments intersect, 1024
  - for enumerating keys in a range, 348 ex.
  - height of, 309
  - insertion into, 315–323
  - joining of, 332 pr.
  - maximum key of, 311
  - minimum key of, 311
  - predecessor in, 311
  - properties of, 308–312
  - relaxed, 311 ex.
  - restructuring, 474 pr.
  - rotation in, 312–314
  - searching in, 311
  - successor in, 311
  - see also* interval tree, order-statistic tree
- REDUCE, 807 pr.
- reduced-space van Emde Boas tree, 557 pr.
- reducibility, 1067–1068
- reduction algorithm, 1052, 1067
- reduction function, 1067
- reduction, of an array, 807 pr.
- reflexive relation, 1163
- reflexivity of asymptotic notation, 51
- region, feasible, 847
- regularity condition, 95
- rejection
  - by an algorithm, 1058
  - by a finite automaton, 996
- RELABEL, 740
- relabeled vertex, 740
- relabel operation, in push-relabel algorithms, 740, 745
- RELABEL-TO-FRONT, 755
- relabel-to-front algorithm, 748–760
  - phase of, 758
- relation, 1163–1166
- relatively prime, 931
- RELAX, 649
- relaxation
  - of an edge, 648–650
  - linear programming, 1125

- relaxed heap, 530
- relaxed red-black tree, 311 ex.
- release time, 447 pr.
- remainder, 54, 928
- remainder instruction, 23
- repeated squaring
  - for all-pairs shortest paths, 689–691
  - for raising a number to a power, 956
- repeat**, in pseudocode, 20
- repetition factor, of a string, 1012 pr.
- REPETITION-MATCHER, 1013 pr.
- representative of a set, 561
- RESET, 459 ex.
- residual capacity, 716, 719
- residual edge, 716
- residual network, 715–719
- residue, 54, 928, 982 pr.
- respecting a set of edges, 626
- return edge, 779
- return**, in pseudocode, 22
- return instruction, 23
- reweighting
  - in all-pairs shortest paths, 700–702
  - in single-source shortest paths, 679 pr.
- rho heuristic, 976–980, 980 ex., 984
- $\rho(n)$ -approximation algorithm, 1106, 1123
- RIGHT, 152
- right child, 1178
- right-conversion, 314 ex.
- right horizontal ray, 1021 ex.
- RIGHT-ROTATE, 313
- right rotation, 312
- right spine, 333 pr.
- right subtree, 1178
- rod-cutting, 360–370, 390 ex.
- root
  - of a tree, 1176
  - of unity, 906–907
  - of  $\mathbb{Z}_n^*$ , 955
- rooted tree, 1176
  - representation of, 246–249
- root list, of a Fibonacci heap, 509
- rotation
  - cyclic, 1012 ex.
  - in a red-black tree, 312–314
- rotational sweep, 1030–1038
- rounding, 1126
  - randomized, 1139
- row-major order, 394
- row rank, 1223
- row vector, 1218
- RSA public-key cryptosystem, 958–965, 983
- RS-vEB tree, 557 pr.
- rule of product, 1184
- rule of sum, 1183
- running time, 25
  - average-case, 28, 116
  - best-case, 29 ex., 49
  - expected, 28, 117
  - of a graph algorithm, 588
  - and multithreaded computation, 779–780
  - order of growth, 28
  - rate of growth, 28
  - worst-case, 27, 49
- sabermetrics, 412 n.
- safe edge, 626
- SAME-COMPONENT, 563
- sample space, 1189
- sampling, 129 ex., 179
- SAT, 1079
- satellite data, 147, 229
- satisfiability, 1072, 1079–1081, 1105, 1123–1124, 1127 ex., 1139
- satisfiable formula, 1049, 1079
- satisfying assignment, 1072, 1079
- saturated edge, 739
- saturation push, 739, 745
- scalar flow product, 714 ex.
- scalar multiple, 1220
- scaling
  - in maximum flow, 762 pr., 765
  - in single-source shortest paths, 679 pr.
- scan, 807 pr.
- SCAN, 807 pr.
- scapegoat tree, 338
- schedule, 444, 1136 pr.
  - event-point, 1023
- scheduler, for multithreaded computations, 777, 781–783, 812
  - centralized, 782
  - greedy, 782
  - work-stealing algorithm for, 812
- scheduling, 443–446, 447 pr., 450, 1104 pr., 1136 pr.
- Schur complement, 820, 834

- Schur complement lemma, 834
- SCRAMBLE-SEARCH, 143 pr.
- seam carving, 409 pr., 413
- SEARCH, 230
- searching, 22 ex.
  - binary search, 39 ex., 799–800
  - in binary search trees, 289–291
  - in B-trees, 491–492
  - in chained hash tables, 258
  - in compact lists, 250 pr.
  - in direct-address tables, 254
  - for an exact interval, 354 ex.
  - in interval trees, 350–353
  - linear search, 22 ex.
  - in linked lists, 237
  - in open-address hash tables, 270–271
  - in proto van Emde Boas structures, 540–541
  - in red-black trees, 311
  - in an unsorted array, 143 pr.
  - in Van Emde Boas trees, 550
- search tree, *see* balanced search tree, binary search tree, B-tree, exponential search tree, interval tree, optimal binary search tree, order-statistic tree, red-black tree, splay tree, 2-3 tree, 2-3-4 tree
- secondary clustering, 272
- secondary hash table, 278
- secondary storage
  - search tree for, 484–504
  - stacks on, 502 pr.
- second-best minimum spanning tree, 638 pr.
- secret key, 959, 962
- segment, *see* directed segment, line segment
- SEGMENTS-INTERSECT, 1018
- SELECT, 220
- selection, 213
  - of activities, 415–422, 450
  - and comparison sorts, 222
  - in expected linear time, 215–220
  - multithreaded, 805 ex.
  - in order-statistic trees, 340–341
  - in worst-case linear time, 220–224
- selection sort, 29 ex.
- selector vertex, 1093
- self-loop, 1168
- self-organizing list, 476 pr., 478
- semiconnected graph, 621 ex.
- sentinel, 31, 238–240, 309
- sequence  $(\{\})$ 
  - bitonic, 682 pr.
  - finite, 1166
  - infinite, 1166
  - inversion in, 41 pr., 122 ex., 345 ex.
  - probe, 270
- sequential consistency, 779, 812
- serial algorithm versus parallel algorithm, 772
- serialization, of a multithreaded algorithm, 774, 776
- series, 108 pr., 1146–1148
  - strands being logically in, 778
- set  $(\{\})$ , 1158–1163
  - cardinality  $(|\cdot|)$ , 1161
  - convex, 714 ex.
  - difference  $(-)$ , 1159
  - independent, 1101 pr.
  - intersection  $(\cap)$ , 1159
  - member  $(\in)$ , 1158
  - not a member  $(\notin)$ , 1158
  - union  $(\cup)$ , 1159
- set-covering problem, 1117–1122, 1139
  - weighted, 1135 pr.
- set-partition problem, 1101 ex.
- shadow of a point, 1038 ex.
- shared memory, 772
- Shell's sort, 42
- shift, in string matching, 985
- shift instruction, 24
- short-circuiting operator, 22
- SHORTEST-PATH, 1050
- shortest paths, 7, 643–707
  - all-pairs, 644, 684–707
  - Bellman-Ford algorithm for, 651–655
  - with bitonic paths, 682 pr.
  - and breadth-first search, 597–600, 644
  - convergence property of, 650, 672–673
  - and difference constraints, 664–670
  - Dijkstra's algorithm for, 658–664
  - in a directed acyclic graph, 655–658
  - in  $\epsilon$ -dense graphs, 706 pr.
  - estimate of, 648
  - Floyd-Warshall algorithm for, 693–697, 700 ex., 706
  - Gabow's scaling algorithm for, 679 pr.
  - Johnson's algorithm for, 700–706
  - as a linear program, 859–860
  - and longest paths, 1048

- by matrix multiplication, 686–693, 706–707
- and negative-weight cycles, 645, 653–654, 692 ex., 700 ex.
- with negative-weight edges, 645–646
- no-path property of, 650, 672
- optimal substructure of, 644–645, 687, 693–694
- path-relaxation property of, 650, 673
- predecessor-subgraph property of, 650, 676
- problem variants, 644
- and relaxation, 648–650
- by repeated squaring, 689–691
- single-destination, 644
- single-pair, 381, 644
- single-source, 643–683
- tree of, 647–648, 673–676
- triangle inequality of, 650, 671
- in an unweighted graph, 381, 597
- upper-bound property of, 650, 671–672
- in a weighted graph, 643
- sibling, 1176
- side of a polygon, 1020 ex.
- signature, 960
- simple cycle, 1170
- simple graph, 1170
- simple path, 1170
  - longest, 382, 1048
- simple polygon, 1020 ex.
- simple stencil calculation, 809 pr.
- simple uniform hashing, 259
- simplex, 848
- SIMPLEX, 871
- simplex algorithm, 848, 864–879, 896–897
- single-destination shortest paths, 644
- single-pair shortest path, 381, 644
  - as a linear program, 859–860
- single-source shortest paths, 643–683
  - Bellman-Ford algorithm for, 651–655
  - with bitonic paths, 682 pr.
  - and difference constraints, 664–670
  - Dijkstra’s algorithm for, 658–664
  - in a directed acyclic graph, 655–658
  - in  $\epsilon$ -dense graphs, 706 pr.
  - Gabow’s scaling algorithm for, 679 pr.
  - as a linear program, 863 ex.
  - and longest paths, 1048
- singleton, 1161
- singly connected graph, 612 ex.
- singly linked list, 236
  - see also* linked list
- singular matrix, 1223
- singular value decomposition, 842
- sink vertex, 593 ex., 709, 712
- size
  - of an algorithm’s input, 25, 926–927, 1055–1057
  - of a binomial tree, 527 pr.
  - of a boolean combinational circuit, 1072
  - of a clique, 1086
  - of a set, 1161
  - of a subtree in a Fibonacci heap, 524
  - of a vertex cover, 1089, 1108
- skip list, 338
- slack, 855
- slack form, 846, 854–857
  - uniqueness of, 876
- slackness
  - complementary, 894 pr.
  - parallel, 781
- slack variable, 855
- slot
  - of a direct-access table, 254
  - of a hash table, 256
- SLOW-ALL-PAIRS-SHORTEST-PATHS, 689
- smoothed analysis, 897
- \*Socrates, 790
- solution
  - to an abstract problem, 1054
  - basic, 866
  - to a computational problem, 6
  - to a concrete problem, 1055
  - feasible, 665, 846, 851
  - infeasible, 851
  - optimal, 851
  - to a system of linear equations, 814
- sorted linked list, 236
  - see also* linked list
- sorting, 5, 16–20, 30–37, 147–212, 797–805
  - bubblesort, 40 pr.
  - bucket sort, 200–204
  - columnsort, 208 pr.
  - comparison sort, 191
  - counting sort, 194–197
  - fuzzy, 189 pr.
  - heapsort, 151–169
  - insertion sort, 12, 16–20

- k*-sorting, 207 pr.
- lexicographic, 304 pr.
- in linear time, 194–204, 206 pr.
- lower bounds for, 191–194, 211, 531
- merge sort, 12, 30–37, 797–805
- by oblivious compare-exchange algorithms, 208 pr.
- in place, 17, 148, 206 pr.
- of points by polar angle, 1020 ex.
- probabilistic lower bound for, 205 pr.
- quicksort, 170–190
- radix sort, 197–200
- selection sort, 29 ex.
- Shell’s sort, 42
- stable, 196
- table of running times, 149
- topological, 8, 612–615, 623
- using a binary search tree, 299 ex.
- with variable-length items, 206 pr.
- 0-1 sorting lemma, 208 pr.
- sorting network, 811
- source vertex, 594, 644, 709, 712
- span law, 780
- spanning tree, 439, 624
  - bottleneck, 640 pr.
  - maximum, 1137 pr.
  - verification of, 642
  - see also* minimum spanning tree
- span, of a multithreaded computation, 779
- sparse graph, 589
  - all-pairs shortest paths for, 700–705
  - and Prim’s algorithm, 638 pr.
- sparse-hulled distribution, 1046 pr.
- spawn**, in pseudocode, 776–777
- spawn edge, 778
- speedup, 780
  - of a randomized multithreaded algorithm, 811 pr.
- spindle, 485
- spine
  - of a string-matching automaton, 997 fig.
  - of a treap, 333 pr.
- splay tree, 338, 482
- spline, 840 pr.
- splitting
  - of B-tree nodes, 493–495
  - of 2-3-4 trees, 503 pr.
- splitting summations, 1152–1154
- spurious hit, 991
- square matrix, 1218
- SQUARE-MATRIX-MULTIPLY, 75, 689
- SQUARE-MATRIX-MULTIPLY-RECURSIVE, 77
- square of a directed graph, 593 ex.
- square root, modulo a prime, 982 pr.
- squaring, repeated
  - for all-pairs shortest paths, 689–691
  - for raising a number to a power, 956
- stability
  - numerical, 813, 815, 842
  - of sorting algorithms, 196, 200 ex.
- stack, 232–233
  - in Graham’s scan, 1030
  - implemented by queues, 236 ex.
  - linked-list implementation of, 240 ex.
  - operations analyzed by accounting method, 457–458
  - operations analyzed by aggregate analysis, 452–454
  - operations analyzed by potential method, 460–461
  - for procedure execution, 188 pr.
  - on secondary storage, 502 pr.
- STACK-EMPTY, 233
- standard deviation, 1200
- standard encoding ( $\{ \}$ ), 1057
- standard form, 846, 850–854
- star-shaped polygon, 1038 ex.
- start state, 995
- start time, 415
- state of a finite automaton, 995
- static graph, 562 n.
- static set of keys, 277
- static threading, 773
- stencil, 809 pr.
- stencil calculation, 809 pr.
- Stirling’s approximation, 57
- storage management, 151, 243–244, 245 ex., 261 ex.
- store instruction, 23
- straddle, 1017
- strand, 777
  - final, 779
  - independent, 789
  - initial, 779
  - logically in parallel, 778

- logically in series, 778
- Strassen's algorithm, 79–83, 111–112
  - multithreaded, 795–796
- streaks, 135–139
- strictly decreasing, 53
- strictly increasing, 53
- string, 985, 1184
- string matching, 985–1013
  - based on repetition factors, 1012 pr.
  - by finite automata, 995–1002
  - with gap characters, 989 ex., 1002 ex.
  - Knuth-Morris-Pratt algorithm for, 1002–1013
  - naive algorithm for, 988–990
  - Rabin-Karp algorithm for, 990–995, 1013
- string-matching automaton, 996–1002, 1002 ex.
- strongly connected component, 1170
  - decomposition into, 615–621, 623
- STRONGLY-CONNECTED-COMPONENTS, 617
- strongly connected graph, 1170
- subgraph, 1171
  - predecessor, *see* predecessor subgraph
- subgraph-isomorphism problem, 1100 ex.
- subgroup, 943–946
- subpath, 1170
- subproblem graph, 367–368
- subroutine
  - calling, 21, 23, 25 n.
  - executing, 25 n.
- subsequence, 391
- subset ( $\subseteq$ ), 1159, 1161
  - hereditary family of, 437
  - independent family of, 437
- SUBSET-SUM, 1097
- subset-sum problem
  - approximation algorithm for, 1128–1134, 1139
  - NP-completeness of, 1097–1100
  - with unary target, 1101 ex.
- substitution method, 83–88
  - and recursion trees, 91–92
- substring, 1184
- subtract instruction, 23
- subtraction of matrices, 1221
- subtree, 1176
  - maintaining sizes of, in order-statistic trees, 343–344
- success, in a Bernoulli trial, 1201
- successor
  - in binary search trees, 291–292
  - in a bit vector with a superimposed binary tree, 533
  - in a bit vector with a superimposed tree of constant height, 535
  - finding  $i$ th, of a node in an order-statistic tree, 344 ex.
  - in linked lists, 236
  - in order-statistic trees, 347 ex.
  - in proto van Emde Boas structures, 543–544
  - in red-black trees, 311
  - in Van Emde Boas trees, 550–551
- SUCCESSOR, 230
- such that ( $:$ ), 1159
- suffix ( $\sqsupset$ ), 986
- suffix function, 996
- suffix-function inequality, 999
- suffix-function recursion lemma, 1000
- sum ( $\Sigma$ ), 1145
  - Cartesian, 906 ex.
  - infinite, 1145
  - of matrices, 1220
  - of polynomials, 898
  - rule of, 1183
  - telescoping, 1148
- SUM-ARRAYS, 805 pr.
- SUM-ARRAYS', 805 pr.
- summary
  - in a bit vector with a superimposed tree of constant height, 534
  - in proto van Emde Boas structures, 540
  - in van Emde Boas trees, 546
- summation, 1145–1157
  - in asymptotic notation, 49–50, 1146
  - bounding, 1149–1156
  - formulas and properties of, 1145–1149
  - linearity of, 1146
- summation lemma, 908
- supercomputer, 772
- superpolynomial time, 1048
- supersink, 712
- supersource, 712
- surjection, 1167
- SVD, 842
- sweeping, 1021–1029, 1045 pr.
  - rotational, 1030–1038

- sweep line, 1022
- sweep-line status, 1023–1024
- symbol table, 253, 262, 265
- symmetric difference, 763 pr.
- symmetric matrix, 1220, 1222 ex., 1226 ex.
- symmetric positive-definite matrix, 832–835, 842
- symmetric relation, 1163
- symmetry of  $\Theta$ -notation, 52
- sync**, in pseudocode, 776–777
- system of difference constraints, 664–670
- system of linear equations, 806 pr., 813–827, 840 pr.
  
- TABLE-DELETE, 468
- TABLE-INSERT, 464
- tail
  - of a binomial distribution, 1208–1215
  - of a linked list, 236
  - of a queue, 234
- tail recursion, 188 pr., 419
- TAIL-RECURSIVE-QUICKSORT, 188 pr.
- target, 1097
- Tarjan's off-line least-common-ancestors algorithm, 584 pr.
- task, 443
- Task Parallel Library, 774
- task scheduling, 443–446, 448 pr., 450
- tautology, 1066 ex., 1086 ex.
- Taylor series, 306 pr.
- telescoping series, 1148
- telescoping sum, 1148
- testing
  - of primality, 965–975, 983
  - of pseudoprimalty, 966–968
- text, in string matching, 985
- then** clause, 20 n.
- Theta-notation, 44–47, 64
- thread, 773
- Threading Building Blocks, 774
- 3-CNF, 1082
- 3-CNF-SAT, 1082
- 3-CNF satisfiability, 1082–1085, 1105
  - approximation algorithm for, 1123–1124, 1139
  - and 2-CNF satisfiability, 1049
- 3-COLOR, 1103 pr.
- 3-conjunctive normal form, 1082
- tight constraint, 865
- time, *see* running time
- time domain, 898
- time-memory trade-off, 365
- timestamp, 603, 611 ex.
- Toeplitz matrix, 921 pr.
- to**, in pseudocode, 20
- TOP, 1031
- top-down method, for dynamic programming, 365
- top of a stack, 232
- topological sort, 8, 612–615, 623
  - in computing single-source shortest paths in a dag, 655
- TOPOLOGICAL-SORT, 613
- total order, 1165
- total path length, 304 pr.
- total preorder, 1165
- total relation, 1165
- tour
  - bitonic, 405 pr.
  - Euler, 623 pr., 1048
  - of a graph, 1096
- track, 486
- tractability, 1048
- trailing pointer, 295
- transition function, 995, 1001–1002, 1012 ex.
- transitive closure, 697–699
  - and boolean matrix multiplication, 832 ex.
  - of dynamic graphs, 705 pr., 707
- TRANSITIVE-CLOSURE, 698
- transitive relation, 1163
- transitivity of asymptotic notation, 51
- TRANSPLANT, 296, 323
- transpose
  - conjugate, 832 ex.
  - of a directed graph, 592 ex.
  - of a matrix, 1217
  - of a matrix, multithreaded, 792 ex.
- transpose symmetry of asymptotic notation, 52
- traveling-salesman problem
  - approximation algorithm for, 1111–1117, 1139
  - bitonic euclidean, 405 pr.
  - bottleneck, 1117 ex.
  - NP-completeness of, 1096–1097
  - with the triangle inequality, 1112–1115
  - without the triangle inequality, 1115–1116

- traversal of a tree, 287, 293 ex., 342, 1114
- treap, 333 pr., 338
- TREAP-INSERT, 333 pr.
- tree, 1173–1180
  - AA-trees, 338
  - AVL, 333 pr., 337
  - binary, *see* binary tree
  - binomial, 527 pr.
  - bisection of, 1181 pr.
  - breadth-first, 594, 600
  - B-trees, 484–504
  - decision, 192–193
  - depth-first, 603
  - diameter of, 602 ex.
  - dynamic, 482
  - free, 1172–1176
  - full walk of, 1114
  - fusion, 212, 483
  - heap, 151–169
  - height-balanced, 333 pr.
  - height of, 1177
  - interval, 348–354
  - $k$ -neighbor, 338
  - minimum spanning, *see* minimum spanning tree
  - optimal binary search, 397–404, 413
  - order-statistic, 339–345
  - parse, 1082
  - recursion, 37, 88–93
  - red-black, *see* red-black tree
  - rooted, 246–249, 1176
  - scapegoat, 338
  - search, *see* search tree
  - shortest-paths, 647–648, 673–676
  - spanning, *see* minimum spanning tree, spanning tree
  - splay, 338, 482
  - treap, 333 pr., 338
  - 2-3, 337, 504
  - 2-3-4, 489, 503 pr.
  - van Emde Boas, 531–560
  - walk of, 287, 293 ex., 342, 1114
  - weight-balanced trees, 338
- TREE-DELETE, 298, 299 ex., 323–324
- tree edge, 601, 603, 609
- TREE-INSERT, 294, 315
- TREE-MAXIMUM, 291
- TREE-MINIMUM, 291
- TREE-PREDECESSOR, 292
- TREE-SEARCH, 290
- TREE-SUCCESSOR, 292
- tree walk, 287, 293 ex., 342, 1114
- trial, Bernoulli, 1201
- trial division, 966
- triangle inequality, 1112
  - for shortest paths, 650, 671
- triangular matrix, 1219, 1222 ex., 1225 ex.
- trichotomy, interval, 348
- trichotomy property of real numbers, 52
- tridiagonal linear systems, 840 pr.
- tridiagonal matrix, 1219
- trie (radix tree), 304 pr.
  - $y$ -fast, 558 pr.
- TRIM, 1130
- trimming a list, 1130
- trivial divisor, 928
- truth assignment, 1072, 1079
- truth table, 1070
- TSP, 1096
- tuple, 1162
- twiddle factor, 912
- 2-CNF-SAT, 1086 ex.
- 2-CNF satisfiability, 1086 ex.
  - and 3-CNF satisfiability, 1049
- two-pass method, 571
- 2-3-4 heap, 529 pr.
- 2-3-4 tree, 489
  - joining, 503 pr.
  - splitting, 503 pr.
- 2-3 tree, 337, 504
- unary, 1056
- unbounded linear program, 851
- unconditional branch instruction, 23
- uncountable set, 1161
- underdetermined system of linear equations, 814
- underflow
  - of a queue, 234
  - of a stack, 233
- undirected graph, 1168
  - articulation point of, 621 pr.
  - biconnected component of, 621 pr.
  - bridge of, 621 pr.
  - clique in, 1086
  - coloring of, 1103 pr., 1180 pr.

- computing a minimum spanning tree in, 624–642
  - converting to, from a multigraph, 593 ex.
  - $d$ -regular, 736 ex.
  - grid, 760 pr.
  - hamiltonian, 1061
  - independent set of, 1101 pr.
  - matching of, 732
  - nonhamiltonian, 1061
  - vertex cover of, 1089, 1108
  - see also* graph
- undirected version of a directed graph, 1172
- uniform hashing, 271
- uniform probability distribution, 1191–1192
- uniform random permutation, 116, 125
- union
  - of dynamic sets, *see* uniting
  - of languages, 1058
  - of sets ( $\cup$ ), 1159
- UNION, 505, 562
  - disjoint-set-forest implementation of, 571
  - linked-list implementation of, 565–567, 568 ex.
- union by rank, 569
- unique factorization of integers, 931
- unit (1), 928
- uniting
  - of Fibonacci heaps, 511–512
  - of heaps, 506
  - of linked lists, 241 ex.
  - of 2-3-4 heaps, 529 pr.
- unit lower-triangular matrix, 1219
- unit-time task, 443
- unit upper-triangular matrix, 1219
- unit vector, 1218
- universal collection of hash functions, 265
- universal hashing, 265–268
- universal sink, 593 ex.
- universe, 1160
  - of keys in van Emde Boas trees, 532
- universe size, 532
- unmatched vertex, 732
- unsorted linked list, 236
  - see also* linked list
- until**, in pseudocode, 20
- unweighted longest simple paths, 382
- unweighted shortest paths, 381
- upper bound, 47
- upper-bound property, 650, 671–672
- upper median, 213
- upper square root ( $\uparrow\sqrt{\phantom{x}}$ ), 546
- upper-triangular matrix, 1219, 1225 ex.
- valid shift, 985
- value
  - of a flow, 710
  - of a function, 1166
  - objective, 847, 851
- value over replacement player, 411 pr.
- Vandermonde matrix, 902, 1226 pr.
- van Emde Boas tree, 531–560
  - cluster in, 546
  - compared with proto van Emde Boas structures, 547
  - deletion from, 554–556
  - insertion into, 552–554
  - maximum in, 550
  - membership in, 550
  - minimum in, 550
  - predecessor in, 551–552
  - with reduced space, 557 pr.
  - successor in, 550–551
  - summary in, 546
- Var [ ] (variance), 1199
- variable
  - basic, 855
  - entering, 867
  - leaving, 867
  - nonbasic, 855
  - in pseudocode, 21
  - random, 1196–1201
  - slack, 855
  - see also* indicator random variable
- variable-length code, 429
- variance, 1199
  - of a binomial distribution, 1205
  - of a geometric distribution, 1203
- VEB-EMPTY-TREE-INSERT, 553
- vEB tree, *see* van Emde Boas tree
- VEB-TREE-DELETE, 554
- VEB-TREE-INSERT, 553
- VEB-TREE-MAXIMUM, 550
- VEB-TREE-MEMBER, 550
- VEB-TREE-MINIMUM, 550
- VEB-TREE-PREDECESSOR, 552
- VEB-TREE-SUCCESSOR, 551

- vector, 1218, 1222–1224
  - convolution of, 901
  - cross product of, 1016
  - orthonormal, 842
  - in the plane, 1015
- Venn diagram, 1160
- verification, 1061–1066
  - of spanning trees, 642
- verification algorithm, 1063
- vertex
  - articulation point, 621 pr.
  - attributes of, 592
  - capacity of, 714 ex.
  - in a graph, 1168
  - intermediate, 693
  - isolated, 1169
  - overflowing, 736
  - of a polygon, 1020 ex.
  - relabeled, 740
  - selector, 1093
- vertex cover, 1089, 1108, 1124–1127, 1139
- VERTEX-COVER, 1090
- vertex-cover problem
  - approximation algorithm for, 1108–1111, 1139
  - NP-completeness of, 1089–1091, 1105
- vertex set, 1168
- violation, of an equality constraint, 865
- virtual memory, 24
- Viterbi algorithm, 408 pr.
- VORP, 411 pr.
  
- walk of a tree, 287, 293 ex., 342, 1114
- weak duality, 880–881, 886 ex., 895 pr.
- weight
  - of a cut, 1127 ex.
  - of an edge, 591
  - mean, 680 pr.
  - of a path, 643
- weight-balanced tree, 338, 473 pr.
- weighted bipartite matching, 530
- weighted matroid, 439–442
- weighted median, 225 pr.
- weighted set-covering problem, 1135 pr.
- weighted-union heuristic, 566
- weighted vertex cover, 1124–1127, 1139
- weight function
  - for a graph, 591
  - in a weighted matroid, 439
- while**, in pseudocode, 20
- white-path theorem, 608
- white vertex, 594, 603
- widget, 1092
- wire, 1071
- WITNESS, 969
- witness, to the compositeness of a number, 968
- work law, 780
- work, of a multithreaded computation, 779
- work-stealing scheduling algorithm, 812
- worst-case running time, 27, 49
  
- Yen's improvement to the Bellman-Ford algorithm, 678 pr.
- y-fast trie, 558 pr.
- Young tableau, 167 pr.
  
- $\mathbb{Z}$  (set of integers), 1158
- $\mathbb{Z}_n$  (equivalence classes modulo  $n$ ), 928
- $\mathbb{Z}_n^*$  (elements of multiplicative group modulo  $n$ ), 941
- $\mathbb{Z}_n^+$  (nonzero elements of  $\mathbb{Z}_n$ ), 967
- zero matrix, 1218
- zero of a polynomial modulo a prime, 950 ex.
- 0-1 integer programming, 1100 ex., 1125
- 0-1 knapsack problem, 425, 427 ex., 1137 pr., 1139
- 0-1 sorting lemma, 208 pr.
- zonk, 1195 ex.