The Hunger Games: Embodied agents evolving foraging strategies on the frugal-greedy spectrum

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Abstract

In Evolutionary Biology and Game Theory, there is a long history of models aimed at predicting strategies adopted by agents during resource foraging. In Artificial Life, the agent-based modeling approach allowed to simulate the evolution of foraging behaviors in populations of artificial agents embodied in a simulated environment.

In this paper, different sets of behaviors are evolved from a simple setting where agents seek for food patches distributed on a two-dimensional map. While agents are not explicitly playing a game of chicken, their strategies are found on a spectrum ranging from a frugal strategy (aka Dove) to a greedy strategy (aka Hawk). This phenomenon is due to the fact that moving is both a way for the agents to play or go to get away from an unfavorable area of the environment. It is also observed that by moving away, the agents preserve the ecology, preventing the resource from disappearing locally.

Those strategies are shown to be stable if the environment is colonized by one given population. However, post-mortem tournaments among different groups of agents (separately evolved), systematically result in a specific group of agents dominating. The optimal strategy in the simulated tournaments is found to be one with fine-tuned timing for leaving. Further analysis shows how the strategy exploits resources without completely depleting them, producing Volterra-like population tendencies.

Introduction

In nature, animals are known to adopt a broad diversity of strategies to forage for resources (O’brien et al. 1990). While some preserve local ecology, others act more aggressively, leading to the deterioration of the local environment or ecological niche (Odling-Smee et al. 1996, Gyllenberg and Parvinen 2001, Kotler et al. 2002). This forces the group to constantly move to new sectors where the resource is plentiful, inducing a risk linked to the uncertainty related to discovery of those areas. A species may evolve a behavior that leads to its own extinction, a phenomenon called Tragedy of the Commons Hardin (1968), Matsuda and Abrams (1994).

In the field of optimal foraging theory (OFT), many models have been developed to predict the behavior of animals foraging for resource patches (MacArthur and Pianka 1964, Pyke 1984). The Ideal Free Distribution theory (Fretwell and Lucas 1970) predicts that if certain minimal conditions are fulfilled, the distribution of animals among patches will minimize resource competition and maximize fitness. The Marginal Value Theorem (Charnov 1976) describes the strategy that maximizes gain per unit of time in systems where resources decrease over time. As those models generally rely on mechanisms akin to darwinian selection to evolve the behaviors (Werner and Hall 1974), the individuals are assumed to maximize their optimal benefit per cost, in order to maximize their fitness. Different agents may change their optimal behavior in agents. This interaction has been studied extensively in the past for the case of prey-predators models (Huffaker et al. 1963, Glass 1971, Turchin 2003).

In game theory, the war of attrition (Smith 1974) is a game in which two players compete for a unitary resource and the winner is the contestant that is prepared to go on longer. In nature, there are countless examples in which individuals bid amounts of time they are ready to spend in order to get a resource, mostly for resource ownership (Smith 1982). This can also be linked to the snowdrift game (Sugden 2004), where the worst outcome is obtained when both players refuse to give up. In this game, players have to choose one of two strategies, one aggressive/greedy named Hawk and one frugal/cooperative named Dove. The greatest payoff, the temptation $T$ is obtained by an Hawk facing a Dove. However, this comes at a risk, as the worst payoff, the punishment $P$ is obtained by an Hawk facing another Hawk. A Dove will get a reward $R$ when facing another Dove, and the sucker’s payoff $S$ when facing an Hawk. These payoffs are such that $T > R > S > P$, so that Dove is the safest strategy, but Hawk gives the highest possible payoff.

Applied to the problem of foraging, individuals locally play a variant of the war of attrition, in the sense that exploiting the same resource for the same time will end up being disadvantageous to the forager. This can be caused by the resource being depleted beyond a self-regeneration threshold, the resource becoming less rewarding or the foraging itself becoming more costly (Davies et al. 2012). This is similar to...
the law of diminishing returns (Lipsitch et al. 1995), where the animal must find out when it is more beneficial to stay or leave a resource. As such, the potential strategy are also similar to those of Hawk and Dove: one can either bet on the others leaving, which might lead to a depleted resource, or leave beforehand for a hopefully more plentiful spot.

The goal of this research is to investigate the behavior of individuals playing a continuous, spatial variant on the war of attrition or snowdrift (Sugden 2004). We are especially interested in the emergence of behaviors that are in between the expected strategies from the discrete version of the game, and the way those strategies interact, when mixed in a population.

In this paper, we thus make use of agent-based modeling (ABM) with a simplistic setup to investigate foraging behavior. We do not explicitly seek the optimal behavior, but rather interpret the behavioral data generated by modeling optimal adaptations to environmental niches (Seth 2007), and focus on analyzing the interaction of agents adopting different strategies (Stephens and Krebs 1986). Several type of behaviors are evolved, ranging over a spectrum going from frugal to greedy. We found examples of evolved behaviors from any position of the spectrum, forming their own niche. Once those behaviors are brought together, we observe however that a compromise turns out to be the best solution. We also show that this solution corresponds to one of a few populations of agents, that have fine-tuned their timing of leaving the resource patches. This “leaving” behavior with a precise timing allows to exploits resources without completely depleting them, producing Volterra-like population tendencies.

Model

We simulate a population of individuals controlled by neural networks, moving about on a two-dimensional toroidal map (Figure 1). The environment is composed of mostly empty space with a preset number of food patches randomly distributed.

The agents have to forage for food, giving them the energy they need to survive and produce offspring throughout the simulation. Agents movements and decisions are calculated per iteration, which represents the quantum of simulated time. Each iteration, the output of the neural network of an agent, as well as its position and energy are updated.

Methodology

Agent are embodied in the sense that they have a position in the environment. Overlap is allowed: it is possible for two agents to occupy the exact same spot. Agents have an internal amount of energy that is depleted over time. This energy can be increased by staying on food patches, up to a fixed maximum. If the internal energy of the agent reaches zero or below, the agent “dies” and is removed from the simulation. Agents also die once they lived until their maximum

Figure 1: Simulation map. Every agent is represented by a small circle, with color representing its genotype (i.e. a vector encoding the weights of its neural network) and color intensity representing its current energy level. The large red circles represent the resource patches, where lighter color indicates that the state of depletion of the patch.

age (1000 iterations). This approach is aimed at favoring the apparition of new genotypes and behaviors.

Agents take actions based on the output of their neural controller, which is implemented with an Elman artificial neural network (Elman 1990) with an architecture in three layers, similar to (Witkowski and Ikegami 2014). The network consists of two input units (encoding the current energy of the agent and the amount of energy received at this iteration), fully connected to two hidden units, themselves fully connected to two outputs units (deciding their steering angle and speed).

Agents are thus not directly playing the snowdrift game in the classic sense. Nonetheless, their position is a form of play, since they can choose how long they will remain on a food patch, or how often they will move.

All nodes in the neural network take activation values between 0.0 and 1.0. All output values are also floating values between 0.0 and 1.0, the first motor output is then converted to an angle between $-\pi$ to $\pi$, and the second motor output converted to a speed factor multiplying the velocity. The activation state of internal neurons is updated according to a sigmoid function.

The network’s weights are evolved following a similar algorithm to previous work (Witkowski and Aubert 2014): each agent, when it reaches a given energy level, produces an offspring. Each weight in the offspring’s network is mutated with the rate given in Table 1. The offspring will start with a set initial energy, equal to the energy lost by the parent. Reproduction is always asexual, with only one parent.

Energy gathering

Food patches provide energy to the agents that are on top of it. However, overfeeding leads to a depletion of a patch’s internal energy. Energy is stable if there is only one agent, decreasing if there are multiple agents, and slowly recovering if no agent is present. Each agent receives a reward equal to maximum reward per patch times the current energy fraction of the patch. Note that for a patch more that 50% depleted, agents are actually losing energy overall. It is also possible
for two patches to overlap, in which case agents get the sum of all rewards. As mentioned above, agents reproduce when their total energy reach the reproduction value, which can thus be considered a soft maximum.

Depletion rate per agent and recovery speed are shown in Table 1. If a food patch is completely depleted, it is destroyed and a new patch is generated at a random position. This keeps the total number of patches identical throughout the simulation.

A typical example of energy consumption over time, by an agent on a patch, is shown in Figure 2. The energy values are here arbitrary. The energy intake starts dropping when more than one individual are on the same patch. The optimal time spent on a patch, maximizing the overall ratio between resource intake and time spent foraging and traveling, can be visualized by connecting the average transit time on the x axis tangentially to the cumulative resource intake (see Figure 3). The optimum is however expected to change over time based on the interaction with other agents, which will be dependent on evolutionary dynamics.

**Experimental setups**

The experiment is separated in three stages. First, in the training phase, agents neural controllers are independently evolved, and the surviving agents are selected for the next phase. Second, in the analysis phase, these strategies are analyzed. Thirdly, in the tournament phase, the resulting agents are evaluated against each other.

At the beginning of the first phase, in each simulation, the world is populated with 500 random individuals. This population is evolved for 5000 iterations, which yields in most cases a uniform population (in the sense that all agents alive have a relatively close common ancestor). The last 5 generated individuals are then stored for the next phase. This approach allows us to get a sampling of the evolved strategy in the run. This was repeated over 6 runs to gather a total of 30 agents. While the sampling might be insufficient to capture completely the strategy of a given population, it was enough to gather a variety of behaviors.

During the second phase, mutations were disabled and the world was seeded with one agent at a time. Agents behavior was categorized by hand and then linked to metrics from runs. Those behaviors are detailed in the next section.

Finally, in the tournament phase, in order to evaluate all the strategies the agents evolved, we performed 100 runs seeded with the 30 individuals. Mutations were again prevented to ensure that the strategy is kept intact throughout the run. This is akin to dilating the time scale from the “evolutionary” scale to a sort of instantaneous interaction scale. This was done in particular to protect highly tuned strategies that does not resist well to mutation.

As a complement, we also performed the same tests where agents were allowed to evolve (i.e. where mutation was active, allowing for progressive change in the behavior of newborn agents with respect to their parents’), which is similar to introducing all species in a common environment. In this case, the robustness of the strategy and its potential adaptability was the paramount factor, yielding slightly different results.

**Results**

In simulations, a full spectrum of behaviors have emerged. While they all lead to agents to consuming the food resource, the level of selfishness varies, ranging anywhere between frugal (dove) and greedy (hawk) strategies. The most common approaches observed are listed below, in increasing order of greediness. The distribution of agents among those

![Figure 2: Typical energy intake over time for an agent on a patch. The number \( n \) of agents on the same patch fluctuates over time. When this number is higher than one, the patch gets depleted, in turn leading to a lower amount of food dispensed in total by that patch.](image1)

![Figure 3: Cumulative resource intake and optimal time to leave a patch. This diagram illustrates the optimal point to leave a resource patch in order to maximize the amount of energy gathered per time spent foraging and traveling, by connecting the average transit time (arbitrarily 25 iterations in this example) on the horizontal time axis tangentially to the cumulative resource intake. The resulting optimal time to leave a patch is therefore 20 iterations.](image2)
The previous behaviors can be characterized in two ways: looking either at the overall movement of the agents or at the strategies implemented by agents. Note that, due to mutations, an agent can have a different strategy than the species it belongs to.

**Butterfly**  Those agents are not staying long in a given place. They tend to gather energy for a few iterations, then move to another patch in straight line. Overtime, agents tend to accumulate enough energy to make children, making this strategy viable. The overall amount of generated offspring is low, and mortality rate is high. This strategy can thus be outperformed by most other. However, in a setting where multiple aggressive strategies are competing against each other, butterflies are mostly unaffected, and can sometimes weather the fight. In this case, they remain the last species standing.

**Circle**  Those individuals are staying close to a given food patch, but making big circles so that only a fraction of their time is spent on the food patch. This strategy keeps the total number of agent on the patch.

**Explorer**  In this case, agents find new food patches, stay long enough to reproduce. Once the energy level of the food patch starts decreasing, they move on to the next spot. This strategy can be considered the “average” of the spectrum.

**Spore**  With this approach, agents colonize a patch, reproduce until near exhaustion, and then massively spread at once when the remaining energy is not enough to offset the leaving cost. This strategy yields “bursting” events, similar to the release of spores or viruses. These spores then populate the nearby food patches and repeat the process.

**Static**  While this is not a viable strategy for a whole species, or even for a group, a few agents evolved this behavior. Those agents typically belong to species with behaviors on the Hawk side of the spectrum, that is, behaviors that favor staying on food patch as long as possible. Pushed to the extreme, such strategies will prevent agents from moving even once the spot has been completely depleted, which leads to the agents’ death. However, during their lives, they may produce offspring with a viable strategy, closer to that of the rest of the species they belong to.

Additionally, it is possible that they will prevent the invasion of a species with a more frugal approach. Indeed, other agents will tend to leave an area with mostly depleted food patches. In that sense, the static strategy is detrimental in a uniform population, but can arguably help ensure to an extent the survival of their species in a mixed environment. This strategy might also have an interest in our particular setting, since completely depleting a food patch will create a new, full, food patch somewhere else in the environment.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>World height</td>
<td>800</td>
</tr>
<tr>
<td>World width</td>
<td>800</td>
</tr>
<tr>
<td>Initial pop (normal run)</td>
<td>500</td>
</tr>
<tr>
<td>Initial pop (tournament)</td>
<td>30</td>
</tr>
<tr>
<td>Max population</td>
<td>10000</td>
</tr>
<tr>
<td>Starting energy</td>
<td>100</td>
</tr>
<tr>
<td>Reproduction energy trigger</td>
<td>200</td>
</tr>
<tr>
<td>Reproduction cost</td>
<td>100</td>
</tr>
<tr>
<td>Existence cost</td>
<td>1</td>
</tr>
<tr>
<td>Food patches</td>
<td>200</td>
</tr>
<tr>
<td>Patch size</td>
<td>20</td>
</tr>
<tr>
<td>Maximum reward per patch</td>
<td>2</td>
</tr>
<tr>
<td>Patch recovery per iteration</td>
<td>0.25</td>
</tr>
<tr>
<td>Energy decrease per agent</td>
<td>0.04</td>
</tr>
<tr>
<td>Maximum patch energy</td>
<td>50</td>
</tr>
<tr>
<td>Maximum speed</td>
<td>10</td>
</tr>
<tr>
<td>Maximum age</td>
<td>1000</td>
</tr>
<tr>
<td>Mutation rate</td>
<td>10%</td>
</tr>
<tr>
<td>Mutation factor</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 1: Simulation parameters. Note that we implement a maximum population, but that, with the current settings, this limit is never reached.

Table 2: Distribution of agents from 6 runs over the various evolved strategies.

<table>
<thead>
<tr>
<th>Butterfly</th>
<th>Circle</th>
<th>Explorer</th>
<th>Spore</th>
<th>Static</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>5</td>
<td>8</td>
<td>7</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 3: Winning rates of both agents over 100 runs. No other winner was observed.

<table>
<thead>
<tr>
<th>Individual 16</th>
<th>Individual 17</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>76</td>
</tr>
</tbody>
</table>

Figure 5: Cumulative stay on a given food patch for a wide variety of strategies. Frugal strategies are characterized by a succession of small peaks, while strategies on the Hawk side of the spectrum favor only one spot for extended period of time, leading to a few very large peaks.

The first point can be highlighted by tracking agents over a few iterations (Figure 4). This allows us to see the path taken by agents, showing strong variations among strategies, as mentioned in their respective descriptions.

The second approach, dubbed cumulative stay analysis, was realized on a run starting with 30 independent agents. These agents were selected from the uniform population of multiple runs which evolved different strategies. We took five representatives of each of those runs and used them to seed the population. The cumulative time spent on a food patch by those agents is shown in Figure 5.

Note that we did not take into account the number of offsprings generated by the different strategies. Instead we focused on the 30 sampled agents and the qualitative difference in their behaviors. Since these behaviors are dependent on being part of a population, we disabled the mutations, so that all agents ran during a specific analysis would have the same strategy.

Tournament

In a first attempt to compare strategies, we seed the world with the 30 individuals sampled from the training phase. To ensure that those strategies do not drift during the evaluation, mutations were disabled. While artificial, this approach is similar to changing the time scale: here, mutations can be considered so slow that they do not happen over the course of the evaluation. This can be seen in biology, for instance with bacteria where the time-scale of evolution of strategies and the scale of using those strategies is widely different (Kerr et al. 2002).

When seeded with the sampled agents, the world is quickly overrun by one of two possible agents, dubbed Individual 16 or Individual 17. Since mutations are disabled, a given "species" is thus only comprised of copies of those very agents. Around 10000 iterations, only one set remains. The winning rates over 100 runs are shown in Table 3. We could not observe any other agent achieving a full population overrun, that is, reaching a state where all live agents are a copy of itself. Neither could we find stable mixed populations, even within evolutionary time much shorter than that needed for genetic drift to leave only one species in control experiments.

Figure 6 depicts the phylogeny of a typical tournament run. At the center of the plot is the root of the tree, corresponding to time zero in the simulation, with 30 initial branches. As these branches progress outward, they ramify into each agent's successive generations of offspring. The time scale is preserved, totaling 20000 iterations. Every fork corresponds to one parturition, with the newborn forking clockwise and the parent counterclockwise.

Strategy analysis

To explain the overwhelming dominance of one genotype over all the others, in each simulation, we take a close look at the details of the behavior it generates. By the previous analysis, they would be categorized as "circle", although they show a behavior closer to the "explorer" strategy when the food becomes scarce. In both cases, multiple copies of the agents can be supported by a given food patch, since they are never on the patch for long and end up taking turns. Once the food is nearly depleted, they actively look for a fresher patch, giving time for the patch to regrow. It is possible to recognize this behavior in agents simply by observing the outputs of their neural network (see Figure 7). Typical inputs and outputs of Individual 17 over its life time are shown in Figure 8. Other agents are left with two choices: either stay longer, making the effort to completely destroy the patch, or leaving earlier, leaving more food for those who remain. As such, it seems that Individuals 16 and 17 have simply evolved an efficient patch finding strategy, and tuned their decision parameters to leave at the most appropriate time.

Predator-Prey oscillations of Individual 17

Since Individual 17 does not consume completely food patches, and

\[1\] These numbers are based on their index in the seed.

even ignores those that are nearly depleted, we can observe Volterra-like oscillations\(^2\) in its population (Figure 9). The difference comes from the fact that the regrowth of the "prey" (the food patches) is linear in time, instead of the usual autocatalytic, and thus exponential, generation. Runs are spread in phase space, due to the time it takes to the population to get homogeneous. However, amplitude and frequency are roughly uniform across runs. This is in part due to the fact that Individual 17 does not destroy food patches, so that the total energy available in the system changes smoothly over time, which mitigates potential irregularities.

**Tournaments with evolution** In the case where agents are allowed to evolve over time (i.e. with a non-zero mutation rate on their genotypes) during the tournament, winning strategies are much more diverse. Over 100 runs, agents with a spore approach now win in a majority of cases, while circles get second best, mostly through Individual 16 and Individual 17 (Table 4). Interestingly, in one instance, a static agent was able to seed the winning population, as its offspring were able to recover a spore strategy through mutation. Note that a winning population expresses a range of behaviors, but that the original strategy remains dominant. As with the tournaments without evolution, in all runs the population is eventually overrun by a species descending from a single original agent.

\(^2\)Lotka (1910), Volterra (1926)

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Table 4: Winning counts in tournaments with evolution, separated by agents and strategies.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Circle</th>
<th>Explorer</th>
<th>Spore</th>
<th>Static</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agent Wns</td>
<td>16</td>
<td>11</td>
<td>17</td>
<td>12</td>
</tr>
</tbody>
</table>

Those results contrast with those obtained without mutation. This may be due to aggressive agents adapting more efficiently to a competitive environment and/or to circle strategies being unstable to mutation.

**Discussion**

The obtained results show the emergence of several sets of behaviors from a simplistic foraging task. All agents evolve a way to find more resource, and some are also found to escape from areas when they become less beneficial. This is similar to Aktipis (2004) where agents outperform more complex strategies by simply walking away from an unfavorable location. The observed behaviors are however richer in complexity than a mere exit strategy, ranging from very static/greedy to more exploring/loose types of motion. On the greedy side, agents are exploiting the resources available to their limit, potentially exploiting the fact that new resources are then created as result. Once a food patch is removed, such agents move to the nearest available spot.
A more conservative approach is to move away from the patch if it is completely drained, giving it time to regrow. The difference between behaviors then lies in the way energy is taken from the patch. The most aggressive of these is to simply gather on top of the patch, getting as much energy for oneself as possible. A more sustainable tactic is to take turns by circling near the patch, which can even allow the patch to recover if necessary. Finally, agents can try to minimize food depletion by taking only a small share of the energy available before moving to another area. As such, those can be seen as spatial and temporal implementation of mixed strategies in the Hawk-Dove game, giving them a physical interpretation.

In all but the most frugal populations, agents have a large impact on the amount of food available in the environment. This leads to oscillatory dynamics as the population grows until passes above the limit that can be sustained by the environment, then loses a number of individuals due to shortage of food, until the resource regrowth made it sufficient again. The amount of resource is therefore limiting the population as a carrying capacity, as described in White (1978).

By evolving agents separately the experiment really isolates populations, artificially evolving them in different ecological niches. Isolation has been hypothesized to help give rise to altruistic behavior (Cohen and Eshel 1976). Indeed, in an isolated population, the individuals have more chance to share common genes with one another, in turn amplifying their tendency to kin selection (Smith 1964), thus resulting in all individuals in a given isolated group adopting the cooperating, Dove-like behavior. When the population is then reintroduced in the initial population, the more efficient, cooperating behavior is susceptible to crystallize to the whole population from an inbred founder effect (Provine 2004, Sapolsky 2004).

Once we reintject different strategies in one environment, where they come into contact and compete with each other, we could observe two favorable approaches, leaning either toward frugal or greedy. While it is understandable that extreme strategies would perform more poorly than more adaptable ones, it is interesting to see that strategies that seem to be the most balanced are also inefficient. Another contribution in this paper is to show that, if we prevent agent populations from modifying their strategies through mutation, only collaborating (Dove-like) strategies remain stable among those two. The reasons can be many, ranging from an effective tuning of the strategy to the environment, making further mutation deleterious, to an ease to perform well in a variety of situations without further adjustment. On the other hand, since spore strategies tend to produce a large amount of offspring while food is available, those strategies may evolve faster, taking better advantage of mutations.

In future work, it would be interesting to investigate a much larger sampling of agents for the tournament. A preliminary test with 800 initial agents seems to show more diversity among victorious agents, with or without mutation of the genotype.

It might also be fruitful to investigate the impact of the density of food patches. As a negative control, agents going in straight line, but able to control their speed were able in 6 runs out of 10 to evolve a stable population, albeit much lower than that of normal agents. Decreasing the density to a level were lucky solutions are not available might change the distribution of evolved strategies, potentially favoring the more frugal ones. Additionally, generating an equivalent payoff matrix from those settings may give insights on the proportions of the various strategies, as well as the overwhelming winning rates of agents 16 and 17 in the tourna-

Figure 8: Typical outputs of Individual 17 over its lifetime. Those are interpreted to represent its angle and speed. The inputs leading to those decisions, the current energy and the current reward \( dE \), are also represented. Note that the outputs varies very little over the agent lifetime, indicating that they might be extremely tuned to the present environment. Oscillations in energy show when the agent is circling around a food patch. When the reward of the patch starts to decrease, the agent moves to another patch.

Figure 9: Total population in multiple runs where Individual 17 was the winner. Based on stochastic conditions at the beginning of the run, the oscillations are out of phase. Some runs may also take longer to reach the final amplitude.

ment analysis.

Finally, agents are not sensing each other directly, and more complex behaviors are theoretically possible if a certain mode of signaling was introduced in the model. In terms of cooperation between agents, this would for example allow for mechanisms more complex, as agents may learn to recognize each other. Adding signal to our agents might yield richer cooperation dynamics among separate species with common tactics.

Acknowledgments

This paper was partially supported by Grant-in-Aid for Scientific Research on Innovative Areas (Research Project Number:15H01612). We would also like to thank Dr. Julien Hubert for helpful comments and discussions.

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